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THE

LONDON SCIENCE CLASS-BOOKS

EDITED BY

G. CAREY FOSTER. F.R.S.

AND

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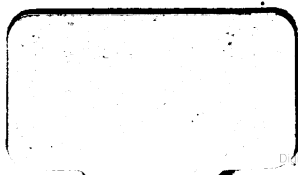


HYDROSTATICS AND PNEUMATICS

BY

SIR PHILIP MAGNUS.

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THE LONDON SCIENCE CLASS-BOOKS

ELEMENTARY SERIES

EDITED BY

PROF. G. C. FOSTER, F.R.S. AND SIR PHILIP MAGNUS

HYDROSTATICS AND PNEUMATICS

HYDROSTATICS AND PNEUMATICS

BY

SIR PHILIP MAGNUS

OF THE CITY AND GUILDS OF LONDON INSTITUTE ;
AUTHOR OF 'LESSONS IN ELEMENTARY MECHANICS,' &c.
JOINT EDITOR OF THIS SERIES

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EDITORS' PREFACE.

NOTWITHSTANDING the large number of scientific works which have been published within the last few years, it is very generally acknowledged by those who are practically engaged in Education, whether as Teachers or as Examiners, that there is still a want of Books adapted for school purposes upon several important branches of Science. The present SERIES will aim at supplying this deficiency. The works comprised in the SERIES will all be composed with special reference to their use in school-teaching ; but, at the same time, particular attention will be given to making the information contained in them trustworthy and accurate, and to presenting it in such a way that it may serve as a basis for more advanced study.

In conformity with the special object of the Series, the attempt will be made in all cases to bring out the educational value which properly belongs to the study of any branch of Science, by not merely treating of its

acquired results, but by explaining as fully as possible the nature of the methods of inquiry and reasoning by which these results have been obtained. Consequently, although the treatment of each subject will be strictly elementary, the fundamental facts will be stated and discussed with the fulness needed to place their scientific significance in a clear light, and to show the relation in which they stand to the general conclusions of Science.

In order to ensure the efficient carrying-out of the general scheme indicated above, the Editors have endeavoured to obtain the co-operation, as Authors of the several treatises, of men who combine special knowledge of the subjects on which they write with practical experience in Teaching.

The volumes of the Series will be published, if possible, at a uniform price of 1s. 6d. It is intended that eventually each of the chief branches of Science shall be represented by one or more volumes.

G. C. F

P. M.

PREFACE.

THIS Class-Book is intended for the use of those pupils in the upper forms of schools who have already acquired some elementary knowledge of the principles of Mechanics, and are about to commence the study of Hydrostatics and Pneumatics.

In the treatment of the subject of this volume I have endeavoured, as far as possible, to combine the Experimental with the Deductive method. Whenever a law is stated, some explanation is afforded of the several experiments by which that law has been established; and whenever a result is deduced, by the aid of mathematical reasoning, from more elementary principles, the pupil is shown how this result may be experimentally verified.

In the hope that this little work may serve as an introduction to more advanced treatises on Hydrostatics, I have devoted a few pages to the consideration of the 'Flow of Liquids through Pipes and Small Orifices'; and, whilst avoiding the mathe-

mathematical difficulties which the fuller treatment of this branch of the subject involves, I have endeavoured to bring into prominence some of the leading principles connected with it, which recent investigations have aimed at establishing.

To facilitate the use of this text-book in class-instruction, the subject-matter is divided into a number of short sections, in which all the more important propositions are illustrated by numerical examples. To nearly every section is appended a set of exercises, progressively arranged, to be solved by the pupil.

My obligations to the published works of different writers are acknowledged in the body of the book.

P. M

LONDON, SAVILE CLUB :

September 1878.

*This volume can be obtained with or without the answers to the
Exercises*

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HYDROSTATICS.

CHAPTER I.

PRELIMINARY NOTIONS.

I. *Nature of Fluid Bodies.*

§ 1. **Matter.**—What we call matter may exist in the solid form as iron, wood, and ice, or in the fluid form as water, oil, air, and steam.

§ 2. **Essential differences between Solid and Fluid Bodies.**—If we take any portion of a solid body, such as a piece of metal, a sheet of glass, or a block of wood, one of the first things we observe is that it possesses a definite shape, which cannot be changed except by the application of a certain amount of force ; we may also observe that it occupies, wherever it may be placed, the same amount of space. If, however, we take a given portion of a fluid substance, such as water or air, we find that it possesses no definite shape, and that it moulds itself to the form of the vessel in which it is contained. Thus if we pour a certain quantity of water from one vessel into another, we observe that whilst the volume of the liquid remains

the same, its shape changes with that of the portion of the vessel which it occupies.

If, again, we endeavour, as in the act of cutting, to separate one part of a solid from another, a certain amount of pressure, depending on the nature of the material, must be exerted ; but if we pass a smooth plane surface, such as the blade of a knife or a sheet of glass, in the direction of its plane, through a mass of fluid, very little resistance is experienced.

These experiments show that the particles of a fluid are *mobile*, i.e., they move freely among one another, and cohere so feebly that they can be separated from one another by the application of a very slight force. We may, therefore, define a *perfect* fluid as a substance the particles of which move freely among one another.

If we take a straight cylindrical tube, open at one end and closed at the other, and fit into it a smooth rod of some solid substance, such as iron or wood, and press the free end of the rod in the direction of its length, the pressure applied is transmitted to the closed end of the tube without producing any effect on its curved surface ; but if the tube contain a fluid instead of a solid, and pressure be applied to it by means of a piston fitting into the tube, the pressure will be felt not only at the base of the vessel, but likewise at all points in its curved surface. This difference, which is characteristic of these two states of matter, is also due to the fact that the particles of a solid, being more or less rigidly connected, are capable of holding together under the influence of a certain amount of force, whilst those of a fluid, being to a

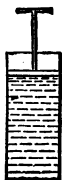
great extent independent of one another, tend to move away in *all* directions when acted upon by a force in any *one* direction. For this reason fluid bodies under the action of gravity cannot rest on a hard horizontal surface, otherwise unsupported, as solids do, but require in addition lateral support. Hence the following definition: Fluid bodies are those which cannot sustain a longitudinal pressure, however small, without being supported by lateral pressure also.

§ 3. **Two kinds of Fluids.**—There are two great classes of fluids, called *liquids* and *gases*, which are thus distinguished :—

If a given quantity of water, which is liquid, be poured into a vessel, it will occupy a certain portion of the vessel and no more ; but if a small portion of hydrogen or carbonic acid, which are gases, be introduced into a vessel, however large, the gas will expand so that some of it will be found in every part of the vessel.

If we take a cylindrical vessel fitted with a piston and fill it with water, we shall find that no amount of pressure we can apply will sensibly diminish the volume of the water ; but if, the vessel being filled with air, we press down the piston, the volume occupied by the gas will be found to diminish as the pressure is increased.

FIG. 1.



Liquids, when submitted to very considerable pressure, have been found to undergo some diminution of volume, but so little, that for most practical purposes they may be considered as incompressible fluids. Hence a *perfect* liquid may be defined as a mass

which is absolutely incompressible and absolutely devoid of resistance to change of shape. Matter, however, satisfying this condition does not exist in nature.

The chief difference between the liquid and the gaseous state of matter consists in this, that whilst a given portion of a liquid has a definite volume, but no definite shape, a given portion of a gas has neither definite volume nor shape, its volume and shape being always the same as that of the vessel containing it.

§ 4. Change from Gaseous into Liquid State.—Some substances exist at ordinary temperatures both in the liquid and gaseous state. Thus we have steam and water, and ether both as a liquid and as a gas or vapour. Other gases cannot be reduced to the liquid state except under the influence of extreme cold and great pressure. The temperature at which the change takes place, and the amount of pressure required, vary considerably. Until very recently, all efforts had failed to reduce certain gases to the liquid condition; and consequently these gases were called *permanent* gases, as distinguished from *vapours* or *liquefiable* gases. But the experiments of MM. Pictet and Cailletet, performed in December 1877, have demonstrated as a fact what was previously only an inference from analogy, that every gas is the vapour of some liquid, and can be reduced to a liquid under the necessary conditions of temperature and pressure. A few days only after M. Pictet of Geneva had succeeded in liquefying oxygen, M. Cailletet of Chatillon-sur-Seine liquefied not only oxygen and carbonic oxide, but likewise hydrogen, nitrogen, and air.

Although there is thus no absolute distinction between permanent gases and vapours, it is convenient to use the word 'vapour' to indicate a gas which at ordinary temperatures can be reduced to the liquid state.

Experiments by Dr. Andrews have shown that gases in changing into liquids can be made to pass through an intermediate condition, in which it is impossible to say to which of these two states of matter they more exactly correspond. By enclosing a vapour in a tube, and subjecting it, at a very high temperature, to a considerable pressure, the vapour can be made to pass by imperceptible degrees, i.e. without any apparent optical change, into the liquid state. For this purpose the vapour must be compressed to that volume which it would occupy in the liquid state, the temperature being sufficiently raised to prevent liquefaction from taking place. At a particular temperature, which is known as the *critical* point, and is different for different gases, the tube is found to be occupied by a homogeneous fluid which is neither a liquid nor a gas, but which changes into one state or the other by slightly lowering or raising the temperature, the volume remaining constant. We thus see that liquids and gases are convertible the one into the other, and that matter can pass from one state to the other without any perceptible gradations.

§ 5. **Viscosity.**—Fluids differ very widely with respect to their distinguishing characteristic, viz. the *mobility* of their particles. In a perfect fluid the particles are supposed to move among one another without encountering any frictional resistance such as

retards the motion of one solid when moving on the surface of another. But no fluid exists which fulfils this condition. By agitating a fluid, that is, by causing one part of it to move against another, heat is generated just in the same way as when two sticks of wood are rubbed together. This shows that the particles encounter frictional resistance to their motion.

Gases approach more nearly to the definition of a perfect fluid than liquids. The latter are found to exhibit every variety of difference with respect to the mobility of their particles, some approaching to a semi-solid condition, and exhibiting properties intermediate between the liquidity of water and the rigidity of ice. Liquids such as treacle, new honey, and tar, in which this frictional resistance appreciably interferes with the mobility of the particles, are called *viscous*. This viscosity, or 'quasi-solidity,' as it is sometimes called, is common to all liquids, and exists, though to a small degree, in water. To this property is mainly due the resistance which a vessel experiences in its passage through the sea ; and it can be shown that a body, completely submerged, and moving with a uniform velocity through a perfect fluid, would experience no resistance whatever to its motion.

§ 6. **The Three States of Matter.**—If we compare together certain typical examples of solid, liquid, and gaseous bodies, such as stone, water, and air, we find that they exhibit distinct and characteristic properties, by which they may be referred to different classes. Keeping these differences in view, we have seen that we can define a *perfect* fluid or a *perfect* liquid, although we know very well that matter nowhere exists

which fulfils the conditions of these definitions. But when we consider the varieties of solid, liquid, and gaseous bodies which experience brings under our notice, we discover that matter actually exists in all intermediate states between these three typical conditions. Thus we have glue-like liquids of every kind, between a clear limpid liquid and a gelatinous or semi-solid mass ; and we have vapours in that critical condition through which they can be made to pass by imperceptible degrees from the gaseous to the liquid state. No strict lines of demarcation can therefore be drawn between these various conditions of matter ; and the solid, liquid, and gaseous states may be regarded as only widely separated forms in which, under different conditions, the same substance may exist.

§ 7. Hydrodynamics defined.—The science which treats of the application of the laws of Dynamics to fluid bodies is generally known as Hydrodynamics. The axioms, or fundamental principles, of the science are Newton's laws of motion, which apply equally to all branches of Dynamics. The mobility of the particles of fluid bodies gives rise to important differences between their behaviour and that of solid bodies under the action of external forces, which render convenient the separate treatment of this subject.

Under the general head of Hydrodynamics are included Hydrostatics and Pneumatics, or the study of the laws of motion in their application to liquid and gaseous bodies respectively.

II. *Units of Measurement.*

In the solution of problems in Hydrostatics a knowledge of the weights of certain volumes of fluid is very generally required. It is desirable, therefore, to note at the outset the different standards of measurement which are commonly employed.

§ 8. **Units of Length.**—The unit of length generally used in England is *one foot*, which is one-third of a yard. The imperial yard is an arbitrary measurement, not derived from any fixed quantity in nature, and is defined ‘as the distance between two marks on a certain metallic bar preserved in the Tower of London, when the whole has a temperature of 60° F.’

In the French or metric system the standard is the *metre*, ‘defined originally as the ten-millionth part of the length of the quadrant of the earth’s meridian from the pole to the equator ; but now defined practically by the accurate standard metres laid up in various depositories in Europe.’ The metre is somewhat longer than the yard, being equal to 1·09362311 yard, or to 39·370432 inches. The great convenience of this system for ordinary purposes is the employment of decimal parts or multiples of the metre, to represent smaller or larger units. Thus, in any expression, if the units represent metres, the *tens* represent *deca*-metres, the hundreds *hecto*-metres, and so on. In the same way the first decimal place represents *deci*-metres, the second *centi*-metres, the third *milli*-metres, and so on. Thus 135·724 metres represents 1 hecto-metre, 3 deca-metres, 5 metres, 7 deci-metres, 2 centi-metres, and 4 milli-metres.

In physical investigations the *centimetre* is now generally accepted as the unit of length. This unit has been selected by a Committee appointed by the British Association, and is recommended for general adoption in a work ¹ published by the Physical Society of London. The system of units, based on the recommendation of this committee, is known as the Centimetre-Gramme-Second system of units, and is generally referred to as the C.G.S. system.

$$1 \text{ foot} = 30.4797 \text{ cm.}$$

When the number of units is very large, it is expressed as the product of two factors, one of which is a power of 10. Thus 3240000000 is written 3.24×10^9 , and 0.00000324 is written 3.24×10^{-6} .

§ 9. **Units of Area.**—In England we use, commonly, the square yard, the square foot, and square inch. In the metric system we have the sq. metre, the sq. decimetre, sq. centimetre, &c.

$$1 \text{ sq. metre} = 100 \text{ sq. decs.} = 10,000 \text{ sq. centimetres.}$$

In the C.G.S. system of units, the unit of area is the square of the unit or length, i.e. 1 sq. centimetre.

$$1 \text{ sq. foot} = 929.01 \text{ sq. cm.}$$

§ 10. **Units of Volume.**—The advantages of the metric system over that ordinarily used in this country are most apparent when we have to compare units of volume with units of length. In our system of measures no simple relation exists between these two units. The gallon, which is the common unit of volume, cannot be represented by any exact number of cubic feet or inches. In the French metric system the unit

¹ 'Illustrations of the C.G.S. System of Units,' by J. D. Everett.

of volume is the *litre*, and a litre is equal to a cubic decimetre. Thus :—

1 cubic metre = 1000 litres.

1 gallon = 4·54346 litres = 277·274 cubic inches.

In the C.G.S. system the unit of volume is the cube of the unit of length, i.e. the cubic centimetre.

1 cubic foot = 28316 cubic centimetres.

§ 11. Units of Mass and Weight.—The British unit of mass is the quantity of matter which weighs one pound. It is defined by standard only.

The French standard is the kilogram, defined originally as the quantity of matter in a litre of water at 4° C., but now practically determined as the mass of a particular piece of platinum preserved in the Ministère de l'Intérieure at Paris, and by standards which have been compared with this.

In the C.G.S. system, the unit of mass is one gram, and is equal to the mass of a unit-volume, i.e. a cubic centimetre of water, at 4° C.

1 grain = ·0647990 grams.

As the weights of bodies are proportional to their masses at places equally distant from the earth's centre, the foregoing units of mass may be taken as equivalent units of weight. Thus the weight of 3 cubic centimetres of water is 3 grams, and the volume occupied by 3,000 grams of water is 3,000 cubic centimetres.

§ 12. Unit of Time; derived Units.—The unit of time is one second. Units of velocity, acceleration, momentum, force, energy, heat, &c., which are based

on the fundamental units of length, mass, and time, are called *derived units*. Thus in the C.G.S. system the unit of force is that force which, acting upon a gram for one second, generates a velocity of one centimetre per second.

§ 13. **Geometrical Relations.**—As problems frequently occur which presuppose a knowledge of the measurements of the areas and volumes of certain figures, the following geometrical relations should be remembered :—

- (1) The ratio of the circumference of a circle to its diameter $= 3.14159 = \frac{355}{113} = \frac{22}{7}$ nearly, and is represented by the Greek letter π .
- (2) The circumference of a circle $= 2 \pi r$, where r is the radius of circle.
- (3) The area of a circle $= \pi r^2$.
- (4) The area of the surface of a sphere $= 4 \pi r^2$.
- (5) The volume or contents of a sphere $= \frac{4}{3} \pi r^3$.
- (6) The area of the curved surface of a cylinder equals the product of the height into the circumference of the base $= 2 \pi r.h$.
- (7) The volume of a cylinder equals the product of the height into the area of the base, $= \pi r^2.h$.
- (8) The area of the curved surface of a cone equals the product of the slant side into half the circumference of the base, $= \pi r \sqrt{h^2 + r^2}$, where h is the height of the cone.
- (9) The volume of a cone equals one-third of the volume of a cylinder on the same base, and of the same height, $= \frac{1}{3} \pi r^2.h$.

III. *Density. Specific Gravity.*

§ 14. **Density.**—If we take two vessels of equal capacity and fill the one loosely with some substance, such as sand, and compress into the other a much larger quantity of the *same substance*, we should say that the *density* of the matter in the one vessel was less than that in the other. If, then, we understand by *mass* quantity of matter, we see that the densities of two bodies of equal volume and of the *same material* are proportional to their masses.

§ 15. **Measure of Density.**—The measure of density is the mass of a unit-volume. If we adopt the cubic centimetre as the unit-volume, and the gram, or quantity of matter in a cubic centimetre of water at 4°C. , as the unit of mass, then the density of a body is measured by the number of grams in a cubic centimetre of its substance ; and if d represent the density of a body whose volume is V and mass M , d is the mass of a unit volume, and $M = Vd$; or $d = \frac{M}{V}$.

§ 16. **Specific Gravity.**—If we take two vessels of equal capacity, and fill the one with mercury and the other with water, we shall find that the one containing the mercury is heavier than the one filled with water. This difference in the properties of the two substances is known as a difference in their *specific gravities*. When we say that lead is heavier than wood, we mean that bulk for bulk the one substance is heavier than the other—that a cubic foot of lead weighs more than a cubic foot of wood.

§ 17. **Measure of Specific Gravity.**—The sp. gr.

of a substance is measured by the weight of a unit-volume of that substance. If d be the mass of a unit-volume, and s its weight, then $s = gd$, where g is the acceleration due to gravity; and if W be the weight of a body whose volume is V and sp. gr. s , then $W = Vs$, or $s = \frac{W}{V}$.

Since, also, $s = gd$, we have $W = gd \cdot V$.

If we adopt the weight of a gram as the unit of weight, the specific gravity of a body is expressed by the weight in grams of a cubic centimetre of its substance.

Ordinary definition of Specific Gravity.—The specific gravity of a substance is very often said to be measured by the ratio of the weight of a given volume of that substance to the weight of an equal volume of some standard substance; and in considering solid and liquid bodies, water at 4°C . is taken as the standard; whilst in the case of gases, air at 0°C . and 76 cm. barometric pressure is employed. But it will be seen that by defining specific gravity as the weight of a unit-volume, we avoid the explicit reference to a ratio, whilst the number expressing the ratio, when water is the standard substance, is the same as the number of grams representing the specific gravity. Thus, if W be the weight of a given volume V of any substance, and W' the weight of the same volume V of water, then according to the ordinary definition $W \div W' = \text{specific gravity}$. But if s be the weight of a unit-volume of the substance, and w the weight of a unit-volume of water, $W = Vs$ and $W' = Vw$. Hence, specific gravity $= \frac{Vs}{Vw} = \frac{s}{w}$, and if we take

the weight of a unit-volume of water to be the unit weight, as we have supposed, then the specific gravity of a substance $= s$, the weight of a unit-volume of that substance.

Since, also, the weight of a unit of mass is equal to g units of force, and is represented in gravitation units by one gram, we see that the numbers representing the densities of bodies represent also their specific gravities, and that the specific gravity of water is unity.

When the specific gravity is considered as a ratio, it is sometimes called the *relative specific gravity*, to distinguish it from the *absolute specific gravity*, or weight of the mass of a unit-volume.

Where, as in the English system of weights and measures, the weight of a unit-volume of the standard substance is not adopted as the unit of weight, the specific gravity of any substance, considered as a ratio, $= s \div w$, where s is the weight of a unit-volume of the substance, and w is the weight of a unit-volume of the standard. If, therefore, S be the *relative specific gravity* of the substance, we have $VS = Vs \div w$ or $VS w = Vs = W$.

Thus, if we wish to find the weight of 4 cubic inches of zinc, we must first know the weight of a cubic foot of water (w), and then the specific gravity of zinc (represented by a ratio) being 7, we have $W = 7 \times \frac{4}{1728} \times w$, and taking w to equal 1,000 oz. roughly, we have $W = \frac{7 \times 4 \times 1000}{16 \times 1728} = 1.01$ lbs. nearly.

The weight of a cubic foot of water is more nearly equal to 62.3 lbs.

In estimating the weights of gases, it is useful to

remember that one cub. cm. of dry air at 0° C. and 76 cm. pressure (at Paris) weighs 0.001293 grams. Under the same circumstances, 13 cubic feet of air weigh very nearly 1 lb. avoirdupois.

§ 18. SPECIFIC GRAVITY OF SOME IMPORTANT SUBSTANCES.

TABLE I.

SOLIDS.

Name of Substance	Specific Gravity	Name of Substance	Specific Gravity
Platinum, cast . . .	20.86	Sulphur, native . . .	2.03
Gold, cast	19.25	Ivory	1.92
Lead, cast	11.35	Graphite	1.8 to 2.4
Silver	10.47	Phosphorus	1.77
Bismuth	9.82	Magnesium	1.74
Copper, hammered . . .	8.88	Amber	1.08
„ wire	8.78	Wax, white	0.97
Brass	8.39	Sodium	0.97
Nickel	8.28	Potassium	0.86
Steel	7.82	Ebony, American . . .	1.33
Iron, wrought	7.79	Oak, English	0.97 to 1.17
Iron, cast	7.21	Mahogany, Spanish . .	1.06
Tin	7.29	Box, French	1.03
Zinc	7.00	Beech	0.85
Antimony	6.71	Ash	0.84
Iodine	4.95	Maple	0.75
Diamond	3.52	Cherry-tree	0.71
Flint-glass	3.78 to 3.2	Walnut	0.68
Aluminium	2.57	Pitch pine	0.66
Bottle-glass	2.60	Elm	0.60
Plate-glass	2.37	Cedar	0.59
Marble	2.84	Willow	0.58
Emerald	2.77	Larch	0.54
Rock-crystal	2.66	Poplar	0.38
Porcelain	2.49 to 2.14	Cork	0.24

TABLE II.
LIQUIDS, AT 0° C.

Name of Substance	Specific Gravity	Name of Substance	Specific Gravity
Mercury	13.596	Water, distilled, at 4° C.	1.000
Sulphuric acid	1.848	Linseed oil	0.940
Nitric acid	1.500	Proof spirit	0.930
Aqua regia	1.234	Olive oil	0.915
Hydrochloric acid . . .	1.218	Ether, hydrochloric	0.874
Blood, human	1.045	Turpentine, oil of . .	0.870
Ale, average	1.035	Brandy	0.837
Milk	1.030	Alcohol, absolute . .	0.796
Sea-water	1.028	Ether, sulphuric . . .	0.720
Vinegar	1.026		
Tar	1.015		

TABLE III.
GASES AT 0° C. AND 76 CM. PRESSURE.

Name of Substance	Specific Gravity	Name of Substance	Specific Gravity
Oxygen	0.001432	Hydrochloric acid gas	0.00164
Atmospheric air . . .	0.001293	Nitrous oxide	0.00197
Nitrogen	0.001267	Carbonic acid	0.00198
Hydrogen	0.000894		
Chlorine	0.003209		

§ 19. To find the density of a combination of two or more substances whose volumes and densities are given.

Let $v_1, v_2, v_3 \dots$ be the volumes of the substances, $d_1, d_2, d_3 \dots$ their respective densities.

Then if V be the volume of the combination, and if no contraction take place,

$$V = v_1 + v_2 + v_3 + \dots$$

and if D be the density of the whole, VD equals the mass of the whole, and therefore,

$$VD = v_1 d_1 + v_2 d_2 + v_3 d_3 + \dots$$

$$\text{or } D = \frac{v_1 d_1 + v_2 d_2 + v_3 d_3 + \dots}{v_1 + v_2 + v_3 + \dots}$$

If, however, as very frequently happens, contraction takes place, and if the volume of the whole is some proper fraction ($= r$) of the sum of the volumes of the parts,

$$V = r (v_1 + v_2 + v_3 + \dots)$$

$$\text{and } D = \frac{v_1 d_1 + v_2 d_2 + v_3 d_3 + \dots}{r (v_1 + v_2 + v_3 + \dots)}$$

A similar proposition holds good if, for density, we substitute specific gravity.

§ 20. To find the specific gravity of a combination of substances, the weights and specific gravities of which are given.

Let $w_1, w_2, w_3 \dots$ be the weights of the components, and $s_1, s_2, s_3 \dots$ their respective specific gravities; then, if W be the weight of the whole and S its specific gravity, we have $W = w_1 + w_2 + w_3 + \dots$

and since $W = VS$, where V is the volume of the whole

$$\frac{W}{S} = \frac{w_1}{s_1} + \frac{w_2}{s_2} + \frac{w_3}{s_3} + \dots$$

supposing no contraction to take place, and therefore

$$S = \frac{w_1 + w_2 + w_3 + \dots}{\frac{w_1}{s_1} + \frac{w_2}{s_2} + \frac{w_3}{s_3} + \dots}$$

But if the volume of the whole be less than the sum of the volumes of the parts in the ratio of $r : 1$, then

$$S = \frac{w_1 + w_2 + w_3 + \dots}{r \left(\frac{w_1}{s_1} + \frac{w_2}{s_2} + \frac{w_3}{s_3} + \dots \right)}$$

The density of a compound can be found in the same way, the masses and densities of the components being given.

§ 21. **Examples.**—(1.) 1,000 cc. of a gas whose density is 12 are mixed with 2,000 cc. of a gas whose density is 16, and the volume of the mixture is diminished by one-third. Find the density of the mixture.

$$D = \frac{1000 \times 12 + 2000 \times 16}{\frac{2}{3}(1000 + 2000)} = 22$$

(2.) Three kilograms of a substance, sp. gr. = 4, are melted with 5 kils. of a substance, sp. gr. = 6, and the volume of the mixture is 0.1 less than the sum of the volumes of its components. Find sp. gr. of mixture.

$$S = \frac{3 + 5}{\frac{9}{10} \left(\frac{3}{4} + \frac{5}{6} \right)} = \frac{8 \times 240}{9 \times 38} = 5\frac{35}{57}$$

EXERCISES. I.

1. Find the weight of a bar of copper, the section of which is 64 square centimetres, and length one metre.
2. Find the size of an iron shot that weighs 60 lbs. *137 cu. in.*
3. Find the weight of a column of mercury 30 inches high, having a uniform section of one square inch. *14.7*
4. A cubic inch of a substance weighs 2 oz. ; find its specific gravity. *3.4*
5. If silver and copper are mixed in the proportion of 2 : 7 by weight, find the specific gravity of the compound *9.19*
6. What mass of copper must be mixed with 200 grams of gold to make the specific gravity of the compound 0.9 of that of gold? *22.4 grams* *21.03 ans*
7. Equal weights of two fluids, of which the specific gravities are s and $2s$, are mixed together, and the mixture occupies three-fourths of the sum of the volumes of its components. Find the specific gravity of the mixture. *163/9*
8. Three litres of a gas, the density of which is unity, are mixed with one litre of a gas density 14, and the mixture occupies half the volume of the original gases. Find its density. *8.5*
9. A bar of cast-iron, sp. gr. = 7.2, is found to weigh 17.5 kils. The length of the bar is 5 decimetres, and sectional area 50 sq. centimetres. Is there a flaw in the casting? If so, what is its size? *69.5 cm*

CHAPTER II.

FLUID-PRESSURE ON SURFACES IMMERSED.

IV. *Explanation of Terms. Pascal's Principle.*

§ 22. **Fluid Pressure.**—The base and sides of a vessel containing a liquid in equilibrium are subjected to a certain pressure, which is caused by the weight of the liquid, and by the resistance which the surface of the vessel offers to the free motion of the liquid.

§ 23. **Intensity of Pressure.**—By intensity of pressure *at a point* is meant the pressure on the unit of area containing that point.

If a small surface a , in contact with a fluid, is maintained in equilibrium by a force P , acting perpendicularly to the surface, then $\frac{P}{a}$ measures the average pressure-intensity at any point of the area ; and if $\frac{P}{a} = p$, then p is the pressure on a unit of area.

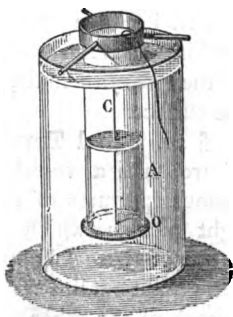
§ 24. **Variation of Pressure with Depth.**—Suppose the small horizontal area a to be at a depth z below the surface of the liquid, then the area a will be pressed vertically downwards by a force equal to the weight of the column of liquid above it ; and if s equal the weight of a unit volume of liquid, this force is equal

to $z a s$. If, then, p be the pressure at any point of this area, $p \cdot a = z a s$, or $p = z s$.

If a is not horizontal, the pressure at some parts of the area is greater than at others. But if we suppose this area to become smaller and smaller, and ultimately to diminish without limit, the difference of pressure at different parts of the area will diminish likewise; and if p represent the pressure intensity, when the area has become so small that it may be regarded as a point, then $p \cdot a = z \cdot a \cdot s$, where a is the infinitely small area about this point, and z is its depth. Hence $p = z \cdot s$, as before, showing that *in the same liquid the pressure varies with the depth*.

This proposition may be experimentally verified by taking a small cylindrical vessel A (fig. 2), open at both ends, and having a movable base o, to which a thread c is attached. If we connect the base with one end of a scale-beam and press it against the vessel by attaching a weight W to the other end of the balance, we shall find, on pouring water into the vessel, that the disc falls off when the water has risen to a certain height, and that this height is always proportional to the excess of the weight W above the weight of the disc. This shows that the ratio of the pressure on the disc to its depth below the surface of the liquid is constant, *i.e.* that the pressure varies with the depth.

FIG. 2.

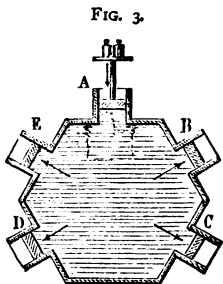


By fixing the vessel A into another vessel containing water, it may be shown that the liquid in the outer vessel exerts an *upward* pressure on the under side of the disc equal to the *downward* pressure of the liquid contained in the inner vessel. For, if we attach the disc o to one end of the scale-beam, and to the other end a weight sufficient to counterbalance it, we shall find, on pouring water into A, that the disc does not fall off until the level of the water in the two vessels is the same.

§ 25. Direction of Pressure on a Surface in Contact.—If a thin plate be immersed in a fluid and be held in any position, the direction of the pressure is perpendicular to the surface of the plate, if the fluid is at rest. For if it acted in any other direction it could be resolved into two components, one perpendicular to the surface and the other along it; and the latter component would, in the absence of friction between the surface and the fluid, produce motion, which is contrary to supposition. The direction of the pressure must, therefore, be perpendicular to the surface.

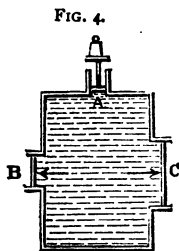
§ 26. Equal Transmissibility of Fluid Pressure. If we take a vessel full of water (fig. 3), having various apertures of the same size, fitted with water-tight pistons which are kept in equilibrium, and if one of these A be pressed downwards with a force P, an additional pressure equal to P will be required at each of the other pistons to preserve equilibrium. Thus if a force of 1 lb. be applied at A, it will be necessary to apply an increased pressure of 1 lb. to each of the other pistons to prevent motion; and as

it matters not in what part of the vessel these pistons are fitted, we see that any increase of pressure applied at one part is transmitted equally throughout the fluid. If the pistons, as in fig. 4, are of different areas, that of B being twice that of A and the area of C being nine times as great, it will require an additional force of 2 lbs. at B, and of 9 lbs. at C, to preserve equilibrium, when a force of 1 lb. is applied at A.

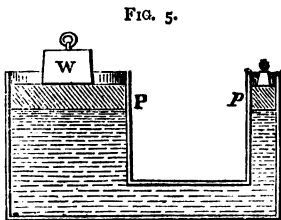


These experiments serve to illustrate the following fundamental law of fluid-pressure, which is known as Pascal's¹ principle :

When pressure is communicated to any part of a fluid, it is transmitted equally in all directions through the fluid.



§ 27. **Mechanical Application.**—The foregoing principle, which is a necessary consequence of the mobility of the particles of a fluid, serves to explain the action of a very useful mechanical contrivance for multiplying power.



¹ Pascal was born at Auvergne, 1623 ; died 1662.

It consists of two communicating vessels containing water, one of which is much larger than the other. The vessels are fitted with pistons P and p , the areas of which we will suppose to be A and a . If now weights W and w be placed on those two pistons respectively, so as to counterbalance one another, it will be found that $W : w :: A : a$, which is in accordance with Pascal's principle.

We see, also, that if the piston p be pressed down through the space S , the water contained in the smaller vessel will pass into the larger, and force up the piston P through some space s , such that—

$$a \times S = A \times s,$$

since the volume of water that is removed from one vessel is the same as that which enters the other vessel. Hence

$$\frac{S}{s} = \frac{A}{a} = \frac{W}{w}$$

or,

$$w \cdot S = W \cdot s,$$

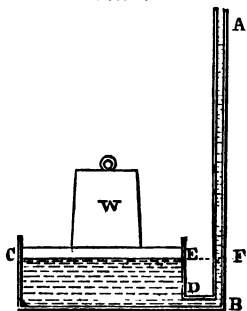
i.e., the work done by w = work done by W .

§ 28. **Hydrostatic Paradox.**—A consequence of Pascal's principle is that a quantity of water, however small, can be made to support a weight, however large, and this seeming paradox can be exhibited in the following manner.

Let AB (fig. 6) be a long narrow pipe communicating with a vessel CD , into which a piston CE is fitted. If now water be poured through the pipe, it will be found to rise to the same level, CE EF , in both parts of the vessel; but if a weight W be placed on CE , and additional water be poured into the pipe,

a position of equilibrium will result such as is shown in the figure. In this case, the pressure exerted by the water in the pipe above the plane $C E F$ supports the weight W . Since this pressure is communicated to every element of area of $C E$ equal to the sectional area of the pipe, the whole pressure on $C E$ is as many times greater than the weight of the water in $A F$ as the area of $C E$ is greater than the area of the pipe.

FIG. 6.



Let a equal the sectional area of the pipe ; then if P is the pressure at F produced by the weight of water above it, $P = a \times A F \times w$, where w is the weight of unit-volume of water, and if A is area of piston $C E$, the pressure communicated to $C E$ is

$$\frac{A}{a} \times a \times A F \times w = A \times A F \times w$$

$$\therefore W = A \times A F \times w.$$

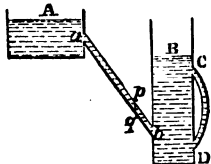
This shows that the weight W can be made as great as we please, by taking $A F$ or $C E$ sufficiently great, and that it is independent of a , the section of the tube. Hence, with a tube sufficiently narrow, a quantity of liquid, however small, can, in principle, support a weight, however large.

§ 29. **Communicating Vessels containing Liquid.** If two vessels A and B (fig. 7) containing the same liquid be connected by a pipe, and if the surface level

of the liquid in A be *higher* than that in B, the liquid will be found to flow from A to B.

This is easily explained by considering the pres-

FIG. 7.



sure on either side of any sectional area, $p q$, of the pipe. For it is evident that the pressure on the side towards A is greater than on the side towards B, since the pressure varies with the depth below

the free surface, and consequently the flow takes place from A to B.

It should be observed that although in the case now considered the liquid flows from A to B, the pressure-intensity along the base of A is *less* than that along the base of B, and also that the pressure at a , where the fluid escapes from the one vessel, is less than that at b , where it enters the other vessel.

If we connect by a tube $c d$ two parts of the same vessel, although the pressure at d is greater than that at c , no flow takes place, for it will be seen that the pressure on either side of any element of area in the pipe $c d$ is the same.

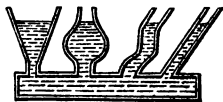
We see, therefore, that a liquid always flows from places of higher to places of lower surface level, and that if a liquid is in equilibrium its surface level must be uniform. Hence it follows that if a liquid be contained in a vessel, its surface must be horizontal; or, more generally, every point of the surface of a liquid in equilibrium must be at the same distance from the earth's centre. In the case of large inland seas, the surface is somewhat curved.

We also see that the pressure at all points in the same horizontal plane within a liquid at rest must be the same, since these points are all equally distant from the free surface, and the pressure varies with the depth (§ 24).

It also follows that if any number of vessels containing liquid communicate with one another, the liquid will stand at the same level in all. For if the surface level were higher in any one vessel, a flow of liquid would take place from that vessel into the others, till the uniformity of surface-level had been established.¹

This fact may be experimentally verified by pouring water into one of the parts of a vessel similar to that shown in the figure, when it will be found that the free surfaces of the liquid in all parts of the vessel lie in the same horizontal plane. The tendency of liquids to find their own level, *i.e.* to flow from places of higher to places of lower surface-level, is of great practical importance. It is utilised in the water-supply of towns. A reservoir of water is kept at a considerable elevation, and from it pipes proceed in all directions conveying water to any heights below the level in which it stands in the main reservoir. The water-level is an instrument which acts on the same principle. It consists of a tube bent at right angles, and furnished at its

FIG. 8.



¹ It should be observed that difference of surface-level corresponds with difference of temperature in heat, and with difference of potential in electricity.

extremities with two glass vessels, and the whole is partly filled with water. Since the liquid has the same surface-level in both branches, any points which the observer's eye detects to be on the same level with the surface of the water in both branches, will be in the same horizontal plane.

When a liquid falls from a higher to a lower level, there is a change of potential into kinetic energy, and a corresponding amount of work can be effected. This fact is economically employed in water mills.

§ 30. **Examples.**—(1.) Find the whole pressure exerted on a horizontal area of 9 sq. cms. which is sunk 125 cms. below the surface of water. The pressure equals the weight of water supported,

$$= 125 \times 9 = 1125 \text{ grams.}$$

(2.) Find the pressure-intensity due to a column of 25 cms. of mercury upon which rests a column of 50 cms. of water.

The pressure-intensity is the pressure per unit area, *i.e.* per sq. cm. :

$$= 25 \times 13.6 + 50 = 390 \text{ grams} = 390 \text{ g units of force,}$$

the specific gravity of mercury being 13.6.

(3.) Find the vertical force necessary to support the horizontal base of a vessel containing mercury, if the area of the base is one square decim., and its depth below the surface of the mercury 5 centimetres, neglecting the weight of the base.

$$P = h a s = 5 \times 100 \times 13.6 \text{ grams.}$$

$$\therefore P = 6800 \text{ grams.}$$

(4.) What must be the height of a column of mercury to exert a pressure of 1220 grams per sq. centim. ?

If h be the height required, $1220 = h \times 13.6$ grams.

$$\therefore h = \frac{1220}{13.6} = 89.7 \text{ cm}$$

§ 31. Equilibrium of two different Liquids in Communicating Tubes.—If two liquids that do not mix meet in a bent tube, or in two tubes communicating with each other, the heights of their free surfaces above their common surface are inversely proportional to their specific gravities.

Let AA' be a plane drawn through the common surface of the two liquids in one of the tubes. Let s and s' be their respective specific gravities. Let B and C be the free surfaces of the liquids. Then if the liquids are in equilibrium, the pressures at A and A' must be the same.

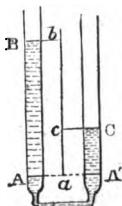
The pressure-intensity at A is $ab \times s$, where ab is the vertical height of B above A .

The pressure-intensity at A' is $ac \times s'$ where ac is vertical height of C above A' .

$$\text{Hence } ab \times s = ac \times s'$$

$$\text{or } ab : ac :: s' : s.$$

FIG. 9.



EXERCISES. II.

1. Two communicating vessels contain fluid, and are fitted with pistons, the diameters of which are 2 inches and 8 inches respectively. If a weight of 3lbs. is placed on the smaller piston, what weight must be placed on the larger to preserve equilibrium?

2. A narrow vertical pipe is attached to a vessel (fig. 6), which is fitted with a piston the area of which is 2 square deci-

metres. If the vessel and pipe contain water, find the height of the water in the pipe when a weight of 40 kils. is placed on the piston.

3. A cylindrical vessel contains mercury to the height of 2 inches above the base, and a layer of 8 inches of water resting on the mercury. Find the pressure at any point in the base, taking sp. gr. of mercury to be 13.6.

4. At what depth below the surface of a lake is the pressure intensity 5 times as great as at a depth of 10 feet, supposing the atmospheric pressure to be equal to the weight of a column of water 34 feet high?

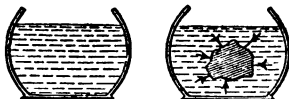
5. Two liquids that do not mix are contained in a bent tube; the difference of their levels is 3 ins., and the height of the denser above their common surface is 5 ins.; compare their specific gravities.

6. If, in the above, the internal section of the tube is one square inch, and the lighter liquid is water; find the weight of water contained in the tube.

V. Whole Pressure on Surface immersed.

§ 32. **Whole Pressure.**—When a vessel contains a fluid, or when a body is immersed in a fluid, the fluid exerts a normal pressure at each point of the sur-

FIG. 10.



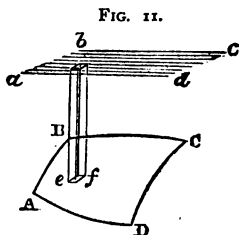
face in contact with it. The sum of all these pressures is called the whole pressure on the surface immersed.

It should be observed that these pressures act in different directions, the pressure at each point being perpendicular to the surface at that point. The whole

pressure is the sum of all these pressures, and represents the total strain to which the vessel containing the fluid, or the body immersed, is exposed.

§ 33. To find the Whole Pressure which a Liquid exerts on a Surface immersed.—Let $A B C D$

be any area in contact with a homogeneous liquid, the free surface of which is the horizontal plane $a b c d$. Let $e f$ be any element of area so small that the pressure-intensity at all points may be supposed to be constant, and let the depth of this area below the



surface of the liquid be z . Then if s be the weight of a unit-volume of the liquid, the pressure exerted on $e f$ is $a z s$, where a is the area of $e f$.

Now the whole pressure is equal to the sum of the normal pressures on all the elements that make up the whole area. Therefore the whole pressure is equal to

$$a_1 z_1 s + a_2 z_2 s + a_3 z_3 s + \dots \\ = (a_1 z_1 + a_2 z_2 + a_3 z_3 + \dots) s.$$

But, by the properties of the centre of gravity,

$$a_1 z_1 + a_2 z_2 + a_3 z_3 + \dots = (a_1 + a_2 + a_3 + \dots) z$$

where z is the depth of the centre of gravity of the whole area below the plane surface of the liquid ;

$$\therefore \text{whole pressure} = A z s,$$

where A is the whole area in contact, z the depth of its centre of gravity below the surface of the fluid, and s the weight of a unitvolume of the liquid. Or, the

pressure = $A \times d \times g$ where d is density of fluid and g the acceleration due to gravity. Hence, *The whole pressure on any area immersed equals the weight of a column of liquid which has that area for base, and the depth of its centre of gravity below the surface of the liquid for height.*

This proposition holds good whatever may be the shape of the vessel containing the liquid ; and whatever may be the position of the area immersed. In some cases the results to which it leads seem at first sight paradoxical, but they will be shown to agree with the general proposition which we have now established.

§ 34. Examples.—(1.) Compare the pressures on the base and side of a cube filled with liquid.

Let the edge of the cube be a , then area of the base is a^2 , and depth of centre of gravity below surface is a ,

\therefore whole pressure on base is $a^2 \times a \times s = a^3 s$.

The area of a side is also a^2 ; but depth of its centre of gravity is

$\frac{a}{2}$, \therefore pressure on side is $a^2 \times \frac{a}{2} \times s = \frac{a^3}{2} s$,

\therefore whole pressure on base = twice the whole pressure of one of the sides.

The pressure on the base acts vertically, and the pressure on the sides horizontally.

(2.) An oblong, the edges of which are 6 metres and 8 metres, is immersed vertically with its shorter edge horizontal, and 2 metres below the surface of water ; find the whole pressure on either side.

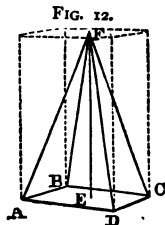
Area immersed is $6 \times 8 = 48$ square metres.

Depth of centre of gravity is $4 + 2 = 6$ metres.

\therefore whole pressure is $48 \times 6 \times s$, where s is the weight of a cubic metre of water, i.e. 1,000 kilograms.

\therefore Pressure = 288,000 kils.

(3.) A right pyramid, the height of which is 8 decs., has a square base, each edge of which is 5 decs. Required the whole pressure on the base when the pyramid is full of liquid. The area of the base is 25 sq. decs., and the depth of its centre of gravity below the surface of the liquid is 8 decs. Hence the whole pressure on the base is $80 \times 2500 \times s = 200s$ kils., where s is the sp. gr. of the liquid. It is to be observed that the pressure on the base, in this case, is greater than the weight of the liquid the vessel contains, being equal to the weight of liquid in the prism having the same base, *i.e.* to a column of liquid the height of which is EF , and base $ABCD$. On the other hand, the pressure transmitted to the stand on which the vessel rests is equal only to the weight of the liquid contained in the vessel; and consequently the pressure on the base of the vessel is greater than the pressure communicated to the stand. This result which follows directly from the general proposition, requires further explanation, and will be considered in the following paragraph.



§ 35. To find the whole pressure exerted by a liquid on the base and sides of the vessel containing it.—We have three cases to consider, according as the sides of the vessel are vertical or slant from the base outwards or inwards. We shall suppose the base of the vessel to be horizontal. If the sides are vertical, as in fig. 13, the pressure on the base is evidently equal to the weight of the fluid contained in the vessel, and the horizontal pressures are equal.

Suppose now that the sides are inclined, outwards in fig. 14 and inwards in fig. 15.

The pressure at any point o in the side AB is a

force p acting perpendicularly to AB . As this force is prevented by the resistance of the slant side from

FIG. 13.

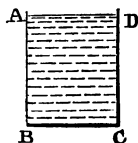


FIG. 14.

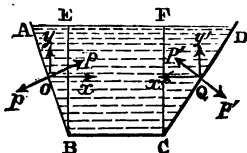
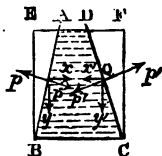


FIG. 15.



causing motion, it produces a reaction equal in magnitude but opposite in direction. This reaction p can be resolved into two components, x and y acting at O , x horizontally in both figs., and y vertically upwards in fig. 14 and vertically downwards in fig. 15. It thus appears that in fig. 14 part of the weight of the liquid is supported by the sum of the forces y acting at all points in the slant side, and the remainder of the weight of the liquid presses on the base BC . Hence, in this case the pressure on the base is *less* than the weight of the contained liquid.

But in fig. 15 the force y acts downwards, and consequently increases the pressure on the base caused by the weight of the liquid; and hence the pressure on the base of the vessel is *greater* than the weight of the contained liquid by the sum of the vertical components of the reactions caused by the pressure of the liquid against all points in the slant sides.

Again, since the force p , due to the pressure of the liquid, and the components, x and y of the reaction of the surface AB , are in equilibrium, they can

be represented by the sides of the triangle AEB . In the same way the forces p', x', y' , acting at Q can be represented by the sides of the triangle DFC .

Hence $p : y : x :: AB : AE : EB$ and

$$p' : y' : x' :: CD : DF : FC$$

But EB is equal to FC , when the base is horizontal $\therefore x = x'$; and in the same way the horizontal pressures at all other points can be proved to be equal. Hence the horizontal components of the pressure of the liquid against the slant side are equal.

If the base is not horizontal, then the whole pressure on the base can be resolved into vertical and horizontal components, and the algebraic sum of the horizontal pressures on the base and sides of the vessel will still be found to equal zero.

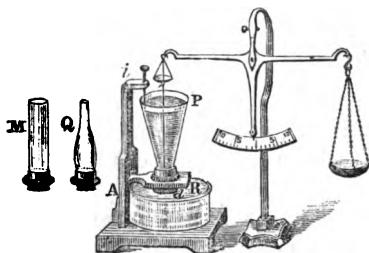
§ 36. **Experimental verification. Pascal's Vases.**

—In order to verify the general proposition, which we have now proved, viz., that the pressure on the base of a vessel containing liquid is independent of the shape of the vessel, and varies only with the area of the base and the depth of its centre of gravity below the surface of the liquid, Pascal contrived an experiment very similar to the following:

Take three vessels P, Q, M , of different shapes, having the same circular aperture at their base, and capable of being screwed into the ring of the stand AB . One of the vessels being fastened to the ring, a circular disc d hanging from one arm of the balance is pressed against it by weights placed in the scale-pan hung to the other arm. Water is then poured into the vessel, and an index i marks the

level at which the water stands when the disc falls off. If the experiment is now tried with the other

FIG. 16.



vessels, it is found that the disc falls off when the water rises to the same level. This shows that the pressure on the base of each vessel is the same when the water is at the same height above it.

EXERCISES. III.

1. Find the whole pressure on a rectangular surface 6 feet by 4 feet, immersed vertically in water with the shorter side parallel to, and 2 feet below the surface.

2. Find the whole pressure on the curved surface of a vertical cylinder which is filled with a liquid, sp. gr. = 1.5, the height of the cylinder being 2 decims., and the radius of the base 7 cms.

3. Show that if a sphere or a cube be filled with liquid the total strain to which it is subjected is three times the weight of the liquid it contains.

4. A flood-gate is 6 feet wide and 12 feet deep. What is the total pressure on the flood-gate when the water is level with the top?

5. A globe, the radius of which is 3.5 cms., rests at the

bottom of a vessel 30 cms. in height, full of water. Find the total pressure on the globe.

6. A conical vessel 2 decimetres high, has a movable base, of 25 sq. cms. area, formed by a disc 2 cms. thick, and the specific gravity of the material being 3.2, find the force necessary to uphold the disc when the vessel is full of water.

7. A smooth vertical cylinder, 2 feet high and 1 foot in diameter, is filled with water, and closed by a piston weighing 3 lbs. Find the total pressure on the curved surface.

VI. Centre of Pressure.

§ 37. **Definition.**—When a plane surface is immersed in a fluid, the pressures at different points of the surface are perpendicular to it (§ 25), and constitute a system of parallel forces of which the whole pressure is the resultant. The magnitude of this resultant pressure we have seen how to find, but the point at which it acts we have not yet determined. This point is called the *centre of pressure*, and may be defined as the *point of action of the single force equivalent to the whole pressure exerted by a fluid on any plane surface with which it is in contact*.

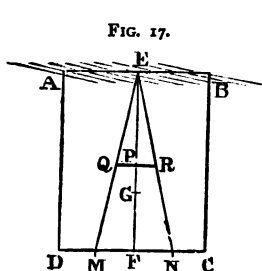
If a plane surface is immersed horizontally the *centre of pressure* corresponds with the *centre of gravity*, but not so if it be immersed in any other position. For in this case the pressures on equal elements of area are not equal, since the pressure varies with the depth (§ 24), and the different elements of area into which the whole surface may be divided are at different depths below the surface of the fluid. Consequently, the centre of pressure, which is the point in the plane surface at which the resultant of

all these forces acts, does not correspond with the centre of gravity of the surface.

The term centre of pressure is used with respect to plane surfaces only, since it is not always possible to find a single force equivalent to the resultant action of a fluid on a curved surface.

§ 38. To find the centre of pressure of a rectangular area immersed vertically.

Let $ABCD$ be a rectangle having its upper side



AB in the surface of the liquid. Then if EF be drawn to bisect AB and DC , the pressure will be equally distributed on each side of EF , and the centre of pressure will lie somewhere in this line.

To determine where, take MN bisected by F to represent the pressure at F . Join EM , EN . Then if we take any point P in EF , and draw QPR parallel to MFN , QR will represent the pressure at P .

For, since the pressure varies with the depth, the pressure at P : the pressure at F :: EP : EF

$$\text{and } EP : EF :: QR : MN$$

$$\therefore \text{the pressure at } P : \text{the pressure at } F :: QR : MN$$

and $\therefore QR$ represents the pressure at P .

Now the problem of finding the point of action of the resultant of a number of forces represented by such lines as QR , acting at all points of EF , is the same as that of finding the centre of gravity of the triangle EMN . But the centre of gravity of this triangle is known to be

at a point G in EF, such that $FG = \frac{1}{3} FE$, and therefore the centre of pressure is at the same point, *i.e.* at a distance of one-third up the middle line from the base.

§ 39. To find the centre of pressure of a triangle having one of its sides in the surface of the liquid.

If the triangle be divided into a number of narrow horizontal strips, the pressure on each strip acts at its middle point, and therefore the *whole pressure* on the triangle acts somewhere in the median line drawn through the middle point of the side in the surface of the liquid. But the pressure on each strip is proportional to its area multiplied by its depth, and the value of this product is constant for every pair of strips at equal distances from the middle point of the median line. Hence the resultant pressure acts at the middle point of this line, or the depth of the centre of pressure is half that of the apex immersed.

§ 40. To find a general expression for the centre of pressure of a plane surface.

Suppose the surface divided by horizontal lines into any number of small elements, then if a be the area of one of these elements and z its depth (which may be considered the same as the depth of its centre of gravity) below the surface of a liquid of sp. gr. s , azs is the pressure on that element of area.

Hence, if z_1, z_2, \dots be the depths of the several elements, the whole pressure equals

$$a_1 z_1 s + a_2 z_2 s + a_3 z_3 s + \dots$$

and if Z be the depth of the centre of pressure, then by the principle of parallel forces

$$\begin{aligned}
 & s (a_1 z_1 + a_2 z_2 + a_3 z_3 + \dots) Z \\
 & \quad = s (a_1 z_1^2 + a_2 z_2^2 + a_3 z_3^2 + \dots) \\
 \text{or } Z &= \frac{a_1 z_1^2 + a_2 z_2^2 + a_3 z_3^2 + \dots}{a_1 z_1 + a_2 z_2 + a_3 z_3 + \dots}
 \end{aligned}$$

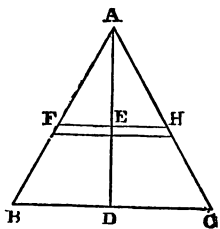
The general application of this method, as of the similar method for finding the centre of gravity of a surface, requires the use of the integral calculus, but special cases may be determined by ordinary algebraical processes.

§ 41. **Examples.**—(1.) To apply the method of § 40 to find the centre of pressure of an isosceles triangle with its apex in the surface of the liquid and its base horizontal.

Suppose the triangle to be divided by horizontal lines into narrow horizontal strips.

Let h = height of triangle, b its base; and let s be the sp. gr. of the liquid.

FIG. 18.



Let h be divided into n parts, then the breadth of each of these strips will be $\frac{b}{n}$. Also,

since $FE : EA :: BD : DA$

$\therefore FE = \frac{b}{2h} AE$, and the area

of each strip may be represented

by $\frac{b}{2h} \cdot AE \cdot \frac{h}{n} = \frac{b}{2n} AE$, and

therefore the pressure on each strip may be taken as equal to $\frac{b}{2n} (AE)^2 \cdot s$, supposing the depth of the strip below A to correspond with that of its centre of gravity. Now since

AE has the several values $\frac{h}{n}, \frac{2h}{n}, \frac{3h}{n} \dots \frac{nh}{n}$, we have

$$\begin{aligned}
 Z &= \frac{\frac{b}{2n} \left\{ \frac{h^3}{n^3} + \frac{(2h)^3}{n^3} + \frac{(3h)^3}{n^3} + \dots \right\}}{\frac{b}{2n} \left\{ \frac{h^2}{n^2} + \frac{(2h)^2}{n^2} + \frac{(3h)^2}{n^2} + \dots \right\}} \\
 &= \frac{h}{n} \cdot \frac{1^3 + 2^3 + 3^3 + \dots}{1^2 + 2^2 + 3^2 + \dots} \\
 &= \frac{h}{n} \cdot \frac{n^2(n+1)^2}{\frac{n^2}{2} + \frac{n^3}{3} + \frac{n}{6}} \quad (\text{by summation of series.}) \\
 &= \frac{h}{4} \cdot \frac{n^3 + 2n^2 + n}{\frac{n^2}{2} + \frac{n^3}{3} + \frac{n}{6}} \\
 &= \frac{h}{4} \cdot \frac{1 + \frac{2}{n} + \frac{1}{n^2}}{\frac{1}{2n} + \frac{1}{3} + \frac{1}{6n^2}} \quad \text{and if } n \text{ be infinite}
 \end{aligned}$$

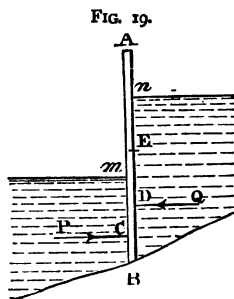
each of the quantities $\frac{1}{n}$ and $\frac{1}{n^2}$ has for its limit zero,

$$\text{Hence } Z = \frac{3}{4} \cdot h.$$

Or, the centre of pressure is on the median line, and at a depth of three-quarters of the height of the triangle, from its apex.

(2.) Required the magnitude and position of the resultant pressure on a flood-gate, the level of the water being different on either side.

Let AB be a section of the flood-gate, and let the height of the water on one side be a , and on the other side b , and suppose b greater than a .



Then, if k be the width of the gate, the total pressure on one side will be—

$$a k \times \frac{a}{2} \times w = \frac{a^2 k w}{2} = P,$$

and on the other side $\frac{b^2 k w}{2} = Q$; and P acts at a point c such that $B C =$ one-third of $B m$, and Q acts at a point D such that $B D =$ one-third $B n$ (§ 38). The resultant of these forces $Q - P$ equals $(b^2 - a^2) \frac{k w}{2}$, and the point E where it acts can be determined by the principle of parallel forces. Thus :

$$\begin{aligned} (b^2 - a^2) \frac{k w}{2} \times B E &= \frac{b^2 k w}{2} \times B D - \frac{a^2 k w}{2} B C \\ &= \frac{k w}{2} \left(b^2 \times \frac{b}{3} - a^2 \times \frac{a}{3} \right) \\ \therefore B E &= \frac{b^2 + b a + a^2}{3(b + a)}. \end{aligned}$$

EXERCISES. IV.

1. A cubical block each edge of which is 5 cm. is sunk in water, with two opposite faces horizontal; find the difference in the pressures on its lower and upper surfaces.

2. A cylinder 10 inches high contains liquid to the height of 8 inches: find the line of action of the total pressure on the interior surface of the cylinder.

3. A hollow cube is three-fourths filled with water. One of the sides of the cube moves freely about a hinge at the base. Required the force that must be applied at the upper edge of the moveable side and perpendicular to it to keep it in equilibrium.

4. Find the height of a cylinder the diameter of which is 2 feet, so that the whole pressure on the curved surface may be four times as great as the pressure on the base, when the cylinder is filled with liquid.

5. A rectangle is immersed in water with one side in the surface. Show how to divide it by a horizontal line into two parts on each of which the whole pressure shall be the same.

6. A rectangular vessel has a partition 10 inches high and 8 inches broad. On one side of the partition is water to the height of 4 inches : on the other side alcohol (sp. gr. 0.8) to the height of 6 inches. Find the magnitude and position of the resultant pressure on the partition.

7. An equilateral triangular lamina is immersed in water with one side in the surface and the opposite angle 2 decimetres below its surface. If each side of the triangle measures 6 decimetres, find the total pressure which the water exerts on it.

8. A rectangle $ABCD$ is immersed in water with the side AB in the surface. Find the pressure on each of the triangles formed by the diagonal AC , and show where it acts. Show also that the resultant of these two pressures coincides with the pressure on the whole rectangle.

9. Find the height to which water may rise on one side of a wall $2\frac{1}{4}$ metres high and half a metre thick without overthrowing it, the specific gravity of the material of the wall being 2.

10. A cylinder, height h inches and diameter $2r$, contains three liquids of specific gravities S_1, S_2, S_3 , which do not mix in layers of equal thickness : find the whole pressure on the base.

CHAPTER III.

FLUID PRESSURE ON BODIES IMMERSED.

VII. *Resultant Vertical Pressure.—Principle of Archimedes.*

§ 42. We have seen (§ 32) that if a body is wholly immersed in a fluid, every point of its surface is subjected to a pressure perpendicular to the surface at that point.

Now, all these pressures, acting as they do in various directions, can be resolved into horizontal and vertical components; and since the horizontal pressures equilibrate each other, the resultant pressure must be vertical, and act upwards or downwards. Moreover, since the pressure varies with the depth, it is clear, speaking roughly, that the whole pressure on the lower half of a body is greater than that on the upper half; and, hence, the resultant of all the pressures on a body immersed is a force acting vertically upwards. This force is called the *resultant vertical pressure*.

§ 43. **Experiment.**—Take an ordinary weight of one pound, and having attached it to a piece of strong thread, let it hang wholly immersed in water from one of the scale-pans of the hydrostatic balance, as shown in fig. 22. It will now be found to weigh less than

1 lb., and this difference of weight is due to the resultant vertical pressure acting upwards.

§ 44. **Measure of the Resultant Vertical Pressure.**—If a symmetrical body with vertical sides be immersed in a fluid, we can easily see what the magnitude of this resultant pressure is. For, since the downward pressure exerted by the fluid on the body is equal to the weight of the column of fluid having A B (fig. 20) for a base, whilst the upward pressure is equal to the weight of the column having c d for a base, the resultant vertical pressure must be equal to the dif-

FIG. 20.

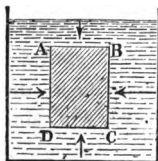
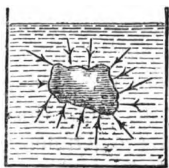


FIG. 21.



ference between the weight of these two columns, *i.e.* to the weight of a column of fluid equal to A B C D, *i.e.* to the weight of the fluid displaced.

If the body be of irregular shape, a more general method of proof must be employed. Thus :

Suppose the body immersed to be a portion of the fluid itself solidified. Then, if no change of density take place, the solidified fluid will remain as before in equilibrium. Hence the weight of this portion of the fluid, which acts, at its centre of gravity, vertically downwards, must be counterbalanced by the upward pressure of the fluid ; and consequently the resultant

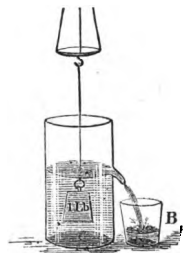
vertical pressure equals the weight of the solidified fluid. But the fluid would exert exactly the same pressure on any other body occupying the same space in the fluid. Hence the resultant vertical pressure on any body immersed is equal to the weight of the fluid displaced, and acts at its centre of gravity, which point is called the *centre of displacement*, or *centre of buoyancy*. The principle thus established is commonly known as the *principle of Archimedes*,¹ and may be thus enunciated :

When a body is immersed in a fluid it is subject to a force equal to the weight of the fluid displaced, which acts at the centre of buoyancy vertically upwards. Or, A body immersed in a fluid loses a portion of its weight equal to the weight of the fluid displaced.

Before proceeding to consider some of the chief deductions from this proposition, we will show how it may be experimentally verified.

§ 45. **Experiments.**—1. Take a vessel with a spout in one side, as shown in fig. 22. Pour in water till it begins to run out from the spout. Take a piece of iron weighing 1 lb., and having suspended it by a fine thread from one of the scale-pans of the hydrostatic balance, weigh it in the water, allowing the water displaced to escape into another vessel, B. The piece of iron will be found to weigh nearly 14 oz., and the weight of the water collected in the vessel B will be found to be a little

FIG. 22.

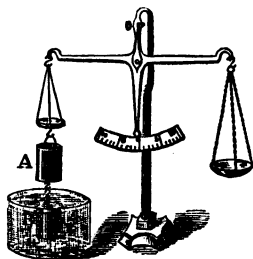


¹ Born at Syracuse in Sicily, flourished about 250 B.C.

more than 2 oz., thus showing that the loss of weight of the body is equal to the weight of the water displaced.

2. Take a hollow brass cylinder, A, into which a solid cylinder, B, exactly fits. Hang the hollow cylinder to one of the

FIG. 23.



scale-pans of the hydrostatic balance, and attach the solid cylinder to it by means of the hook, as shown in fig. 23. Now weigh carefully the two cylinders. Having observed their weight,

take a vessel containing water, and let the solid cylinder hang in it completely immersed. Equilibrium is destroyed, and the free scale-pan descends. Now fill the hollow cylinder with water, and equilibrium is at once restored, clearly showing that the weight of the water in the hollow cylinder, *i.e.* the weight of a quantity of water of the same size as the body immersed, is equal to the loss of weight of the solid in water, *i.e.* to the upward pressure which the liquid exerts on the body.

§ 46. Real and apparent Weight of Bodies.—

We have seen that a body surrounded by a fluid is pressed upwards by a force equal to the weight of the fluid displaced. This is the case with all bodies weighed in air, and consequently their *real* weight (*i.e.* their weight *in vacuo*) is greater than their *apparent* weight by the weight of the air displaced.

If two bodies, the volumes of which are not equal, balance one another in air, their *real* weights are not the same : that which has the greater volume is *really* the heavier. Thus, let the volumes of the two bodies be V and V' , their *real* weights W and W' . Then, if they balance in air, $W - Vs = W' - V's$, where s is the specific gravity of air.

Hence, $W = W' + (V - V')s$, or,
 W is greater than W' , if V is greater than V' .

When the weights of two bodies, of small and nearly equal volumes, are compared, the difference between their real and apparent weights is so slight as to be in most cases of no practical importance. But where the volume of one body is much greater than that of the other, the quantity $(V - V')s$ cannot so easily be neglected. Thus, in answer to the question, Which is the heavier, a pound of feathers or a pound of lead? we should say that the *apparent* weights of both are the same; but the *real* weight of the feathers is greater than that of the lead, and the two would not equilibrate each other in vacuo.

§ 47. **Examples.**—(1.) A solid cube of metal the edge of which is 3 inches, and whose specific gravity is 7, is wholly immersed in water and is supported by a string attached to it. Find its apparent weight in water.

Let W be the weight of the body in air, which differs very slightly from its *real* weight, A its *apparent* weight in water, and Y the resultant vertical pressure acting upwards. Then the body is in equilibrium under the action of these three forces, and, therefore, $W = Y + A$.

Now, the resultant vertical pressure Y equals the weight of

water displaced $= 3^3 \times w$, where w = weight of a cubic inch of water, and $W = 3^3 \times 7 \times w$.

$$\begin{aligned} \text{Hence } A = W - Y &= 3^3 \times 7 \times w - 3^3 \times w = 6 \times 3^3 \times w \\ &= \frac{162 \times 1000}{1728} \text{ oz.} = 5 \text{ lbs. } 13 \frac{3}{4} \text{ oz.} \end{aligned}$$

(2.) A body weighs in vacuo 560 grams, and in water 60 grams: find its volume.

Here $W - A = 560 - 60 = 500$ grams = weight of water displaced. Hence volume of the water displaced, *i.e.*, the volume of the body = 500 cubic centimetres.

(3.) Two hollow spheres, the volumes of which are 100 and 200 cubic centimetres respectively, balance one another in vacuo. What weight must be placed inside the larger that they may balance in water?

Weighed in water the larger sphere will *seem* to be the lighter, since the force supporting it is greater. Now the force supporting the larger sphere in water is 200 grams, and the force supporting the smaller sphere is 100 grams. Hence, for equilibrium, the weight of the larger must be increased by 100 grams without increasing its volume. This may be done by placing 100 grams inside.

(4.) Find the acceleration with which a heavy smooth body will sink in a perfect fluid less dense than itself.

Let W = the weight of the body, s its absolute specific gravity.

Let s' = the specific gravity of the fluid.

Then the volume of the body is $\frac{W}{s}$ (§ 17) and the weight of the fluid displaced is $W \frac{s'}{s}$.

Hence, the resultant force measured in gravitation units, causing the body to descend, is $W \left(1 - \frac{s'}{s} \right)$ and the mass of the body moved is $\frac{W}{g}$.

If, therefore, f is the acceleration with which the body descends, it is shown in books on Mechanics that

$$f = \frac{W \left(1 - \frac{s'}{s}\right)}{W} \cdot g \quad \text{or} \quad f = \frac{s-s'}{s} \cdot g$$

If the body be specifically lighter than the fluid, the body, if placed under the fluid, will tend to rise to the surface, and the acceleration $f = \frac{s'-s}{s} \cdot g$.

EXERCISES. V.

1. Find the weight in water of a piece of zinc (sp. gr. = 7) that weighs 8 oz. in vacuo.

2. Find the volume of a body that weighs 350 grams in vacuo and 225 in water. $350 - 225 = 125 \text{ gm}$

3. A block of stone 2 cubic feet is wholly immersed in water. With what force is it buoyed up? $2 \times 62.5 = 125 \text{ lb}$

4. A piece of metal weighs 36 lbs. in air and 32 lbs. in fresh water. What will it weigh in sea-water the sp. gr. of which is 1.025? $36 - 32 = 4 \text{ lb}$

5. An air-ball, the volume of which is 6 cubic decs., weighs in air 15 grams. Find its *real* weight, having given that the weight of 1 cubic centimetre of air is 0.0013 grams.

6. A round disc of lead (sp. gr. = 11.35) area 5 sq. centimetres and thickness 2 centimetres, is fixed to a piece of cork (sp. gr. = 0.24) of same area and 8 centimetres thick. Find the weight of both in water.

7. The edge of a hollow cube of lead is 8 centimetres, its thickness is 2 centimetres. Find its weight in water. $11.35 \times 448 = 512$

8. A bottle weighing 200 grams is completely filled with water, when it is found to weigh 800 gr. If a piece of iron (sp. gr. = 7.2) is placed in the bottle, the bottle with its contents weighs 955 grams. Find the weight of the iron.

9. A piece of metal of specific gravity 8, and weighing

20 lbs., is dropped into a cylinder *filled* with water. Find the additional pressure on the base.

10. A cylinder of wood (sp. gr. = 0.75), one decimetre high, with a sectional area of 16 sq. cm., is immersed with its axis vertical in alcohol (sp. gr. = 0.8). Find the least weight that must be placed on the top of it to bring its upper surface on a level with the alcohol.

11. A piece of wood weighing 8.25 grams (sp. gr. = 0.66) is placed 4 ft. 7 in. deep in water and is free to rise. Neglecting all frictional resistance, find its velocity when it reaches the surface of the water.

12. Find the time occupied by a stone (sp. gr. = 3.2) in falling from rest through 55 feet of water.

13. Two spheres whose radii are respectively R and r cm., and both of which are heavier than their respective bulks of water, are of equal weight. What weight of metal (sp. gr. = s) must be attached to the *bottom* of the larger that they may balance each other in water?

14. Two masses of given specific gravities balance when suspended from the equal arms of a lever in a known fluid. What is the specific gravity of a fluid in which they balance when one of the masses is doubled?

15. A cubic inch of one of two liquids weighs a grains, and of the other b grains. A body immersed in the first weighs p grains and in the second q grains. What is its *real* weight and what is its volume?

VIII. *Floating Bodies—Metacentre.*

§ 48. When a body is immersed in a fluid, we have to consider three separate cases:—

1. The weight of the body may be *greater* than the weight of the fluid displaced, in which case motion will take place in the direction of the greater force, and the body if left to itself will sink.

2. The weight of the body may be *equal* to the weight of the fluid displaced, in which case it will rest anywhere in the fluid.
3. The weight of the body may be *less* than that of the fluid displaced, in which case the resultant vertical pressure will force the body upwards, and it will float.

These three cases can be easily illustrated by experiments. The first of them has been already considered in the preceding paragraphs, and it has been shown that in order that the body may be prevented from sinking in the fluid, it must be upheld by a force A equal to its apparent weight, such that

$$W = Y + A,$$

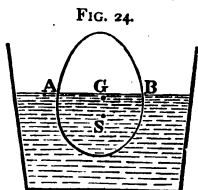
when W is the weight of the body in vacuo, and Y the resultant vertical pressure, or weight of the fluid displaced.

The second occurs less frequently, and does not now need to be separately treated. Later on, when speaking of the formation of drops, we shall have occasion to consider the form assumed by a mass of fluid which is wholly supported by the external resultant pressure.

The third case is of great practical importance, and brings us to the consideration of floating bodies.

§ 49. **Principle of Flotation.**—If we take a body the weight of which is less than the weight of an equal volume of a liquid into which it is immersed, the resultant vertical pressure will bring it to the surface, and the body will be found to assume a posi-

tion of equilibrium in which it will be only partly immersed. If the experiment indicated in § 45 (1) be tried with a body lighter than water, it will be found that the weight of the liquid that escapes in consequence of the partial immersion of the body is exactly equal to the weight of the body itself.



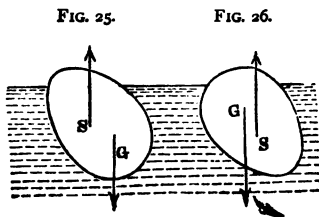
The relation between the part of the body which is immersed and the part that rises above the surface of the fluid is determined by the principle that the two forces acting on the body must equilibrate each other ; *i.e.* the weight of the whole body acting downwards must equal the weight of the fluid displaced acting upwards. Thus if *A B* (fig. 24) be the intersection of the body with the surface of the liquid in which it floats, the position of *A B* is determined by the equation $W = Y$, where W is the weight of the body, and Y that of the liquid displaced. Moreover, if *G* (fig. 24) be the centre of gravity of the body and *s* the centre of buoyancy, *G* and *s* must be in the same vertical line, for otherwise the body would be acted upon by a couple which would produce oscillation or rotation. Hence the conditions of a body floating in perfect equilibrium are :—

1. The weight of the body must equal the weight of the fluid displaced.
2. The centres of gravity of the body and of the fluid displaced must lie in the same vertical line.

§ 50. **Stable and unstable Equilibrium.**—If a floating body be slightly displaced, so that the points

s and G do *not* lie in the same vertical line, the moment of the forces acting on the body may either tend to restore the body to its original position, as in fig. 25, in which case the equilibrium is said to be *stable*, or to overturn the body altogether, as in fig. 26, in which case the equilibrium is *unstable*.

Whether a floating body, partly immersed, can suffer a small displacement without being overturned

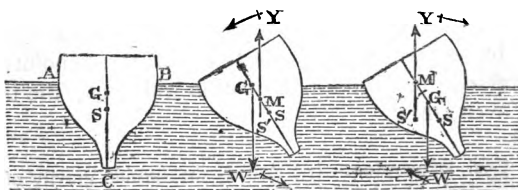


is a matter of the greatest practical importance, as the safety of all vessels at sea depends on it. For, in consequence of the action of the tides and waves, a vessel seldom or never floats in perfect equilibrium, but is continually undergoing a rotatory displacement, which makes it oscillate about its original position of equilibrium. Now we see that if s is above G the body will always return to its original position, or in this case the equilibrium of the vessel may be said to be *stable*. But where s is below G, the body is very likely to overturn if displaced, and hence the danger of overloading the deck, and the advantage of ballast in lowering the centre of gravity of the vessel. But a body may float in stable equilibrium even if s be below G, as we shall see presently ; and consequently

we cannot determine from the relative positions of s and G only whether the equilibrium of a floating body is unstable or not.

§ 51. **Metacentre.**—When a body floating in equilibrium receives a slight displacement, the centre of gravity of the fluid displaced assumes a new position on which the stability of equilibrium mainly depends. Thus let $A B C$ (fig. 27) be a body floating in equilibrium, and let s be the centre of buoyancy. Suppose, now, the body to undergo a slight displacement, in

FIG. 27.



consequence of which the centre of buoyancy changes to s' . If, then, the vertical through s' meets in the point M , the line through s and G which was vertical in the first position of the body, it is clear that when M is above G the equal forces acting on the body will form a couple tending to restore the body to its former position, and that when M is below G they will tend to overturn the body. The point M is called the *metacentre* of the body, and its position depends on the shape of the body and on the position of its centre of gravity. Of course, if G is below s the point M will be above G . Hence it follows generally that the equilibrium of a *floating body is stable or unstable*

according as the metacentre is above or below the centre of gravity of the body.

The metacentre may be defined as the point in which the vertical through the centre of buoyancy of a floating body which has undergone a slight displacement, intersects the line drawn *vertically* through the centre of buoyancy in the original position of equilibrium.

If a body floats *wholly* immersed in a fluid, no change in the position of the body will alter the relative positions of the centres of gravity and buoyancy. Hence in this case the equilibrium cannot be stable unless the centre of gravity is *below* the centre of buoyancy. If the positions of these points be reversed, the slightest oscillation will cause the body to overturn.

§ 52. **Examples.**—(1.) A body the specific gravity of the material of which is s floats in a liquid specific gravity s' . Find what fraction of its volume will be immersed.

Nearly all problems on floating bodies may be solved by the application of the principle of Archimedes, *i.e.* by equating the weight of the floating body with the weight of the fluid it displaces.

Let V equal volume of the body; V' of the portion immersed.

Then Vs = weight of body; and $V's'$ = weight of fluid displaced;

$$\therefore Vs = V's' \text{ or } \frac{V'}{V} = \frac{s}{s'}$$

(2.) A cylinder the height of which is 12 inches floats two-thirds immersed in water. Find what part of it will remain immersed if a liquid the specific gravity of which is 0.2 be poured upon the surface of the water so as to completely cover the cylinder.

When the cylinder floats in the water only, the weight of the cylinder equals the weight of the water displaced.

$$\therefore 12 \times s \times a = 8 \times a$$

where s is the specific gravity and a the sectional area of the cylinder.

$$\therefore \text{specific gravity of cylinder} = \frac{8}{12}.$$

When the lighter liquid is poured on to the surface of the water, the weight of the cylinder equals the sum of the weights of the fluids displaced.

If z equals in this case the part of the cylinder immersed in the lighter liquid we have

$$\begin{aligned} 12 \times \frac{8}{12} &= z \times 0.2 + (12 - z) \\ \text{or } 80 &= 2z + 120 - 10z \\ \therefore z &= 5 \text{ inches.} \end{aligned}$$

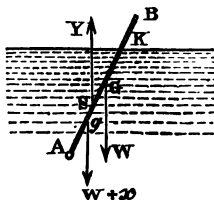
Hence the cylinder rises one inch out of the water in consequence of the additional upward pressure due to the liquid poured into the vessel.

(3.) Required the weight that must be attached to the end of a straight rod of known weight and specific gravity, that it may float vertically in water.

A uniform straight rod cannot float in a vertical position, because the centre of gravity is always above the centre of buoyancy. If, however, a heavy particle of no considerable magnitude be attached to the lower end of the rod, the centre of gravity may be lowered as much as we please. What is required, therefore, is the weight of a particle that will lower the centre of gravity to the depth, at least, of the centre of buoyancy.

Let G be the centre of gravity of the rod AB , a its sectional area, and s its absolute specific gravity, and W its weight. Let x be the weight attached to the end A , and g be the centre of gravity of W and x . Then

FIG. 28.



mined by equating the moments of W and Y about A . Taking the sectional area, which is uniform, as unity we have

$$W = AB \times s \text{ and } Y = BK \times d,$$

where s and d are the specific gravities of the material of the rod and of the liquid respectively. Also

$$W \times GM = Y \times SN \quad \therefore \frac{W}{Y} = \frac{SN}{GM}$$

$$\therefore \frac{AB \cdot s}{BK \cdot d} = \frac{AS}{AG} \text{ by similarity of triangles.}$$

$$\therefore \frac{AB}{AB - AK} \cdot \frac{s}{d} = \frac{AB + AK}{2} \div \frac{AB}{2}$$

$$\therefore \frac{s}{d} = 1 - \left(\frac{AK}{AB} \right)^2$$

$$\therefore \frac{AK}{AB} = \sqrt{\left(1 - \frac{s}{d} \right)} \quad \therefore AK = AB \sqrt{\left(1 - \frac{s}{d} \right)}.$$

If AC is given, AK can be expressed in terms of AC and of the angle BAC , which is thus determined.

EXERCISES. VI.

1. A cylinder of wood (sp. gr. = 0.6) and 12 inches high floats with its axis vertical in water. To what depth will it be immersed?

2. A cube of oak (sp. gr. = 0.97) each edge of which is 6 inches, floats partly in sea water (sp. gr. 1.028) and partly in olive oil (sp. gr. = 0.915). Find what part of it is immersed in each liquid.

3. A uniform block of metal 10 inches high (sp. gr. = 8) floats in mercury (sp. gr. 13.6). Find how much it rises out of the mercury if water be poured on the surface of the mercury so as to completely cover the block of metal.

4. A cylinder floats vertically in a liquid. Compare the forces necessary to raise it and to depress it to an equal extent.

5. A heavy uniform rod weighing 3 kils. and sp. gr. = 6 moves freely under water in a vertical plane about a hinge at one end. If a string tied to the other end supports the rod in a horizontal position, find the tension in the string.

6. A piece of gold (sp. gr. 19.25) weighing 96.25 grams immersed in a vessel full of water causes 6 grams of water to be displaced. Is the gold solid? If not, find the size of the hollow.

7. Two equal globes, the volume of each being 100 c.c., are suspended from the equal arms of a lever, the one hanging completely immersed in water, the other in a liquid of sp. gr. = 0.8. What additional weight is required to make them balance?

8. A cube of metal is floating in mercury (sp. gr. = 13.6). When a weight of 170 lbs is placed on the top, it is observed to sink 3 inches. Find the size of the cube.

9. If an iceberg (sp. gr. = 0.918) float in sea-water, what is the ratio of the part submerged to that which is seen above water?

10. Find what quantity of cork must be attached to a man whose weight is 168 lbs. and sp. gr. = 1.12 so as to enable him just to float in water.

11. An iron ball of 12 lbs. weight floats in mercury covered in water. Find the weights of the parts in the two fluids; having given specific gravity of mercury = 13.6, specific gravity of iron = 7.5.

12. The specific gravities of the upper and lower of two fluids that do not mix are 0.9 and 1.1; the upper fluid is 4 in. deep; a cube with an edge of 1 foot and sp. gr. 0.75 floats in the liquids; how much of it is immersed?

CHAPTER IV.

SPECIFIC GRAVITY, AND MODES OF DETERMINING IT.

IX. *Application of the Principle of Archimedes to the determination of the Specific Gravity of Bodies.*

§ 53. To find the specific gravity of a substance we require to know its weight in vacuo, and its volume, since $\frac{W}{V} = s$. The volume of the body, expressed in cubic centimetres, is numerically equal to the weight of an equal bulk of water at the standard temperature expressed in grams, since the weight of a unit-volume of water is one gram ; and, therefore, the specific gravity of a substance is expressed numerically by the ratio of its weight to that of an equal bulk of water.

Instead of the weight of the body in vacuo, we generally substitute the weight in air, the difference being unimportant for solid and liquid bodies of small magnitude. Where the volume of water equal to that of the body can be directly found, the specific gravity is very easily determined ; but where this is not the case different methods have to be employed.

The principle of Archimedes tells us that a body immersed in a fluid is pressed upwards by a force

equal to the weight of the fluid displaced. Hence the specific gravity of a body can be found by comparing the *real* weight of a body with the difference between its *real* weight and its *apparent* weight in water, since this difference is equal to the weight of water displaced. If, therefore, W represent its real weight, and A its apparent weight in water, the specific gravity of the body equals $\frac{W}{W-A}$.

§ 54. To find the specific gravity of a solid body insoluble in water.

This can be determined roughly by an experiment similar to that described in § 45 (1), where the weight of the body may be directly compared with the weight of the water displaced. But a more accurate result may be obtained by using the hydrostatic balance for finding A , the weight of the body in water. The specific gravity can then be determined by dividing W , the weight of the body, by $W-A$, the loss of weight in water ; or

$$\text{specific gravity} = \frac{W}{W-A}$$

§ 55. To find the specific gravity of a solid body that floats in water.

In this case the body may be said to have a negative weight in water : *i.e.* it requires the application of a positive force to make it weigh zero when completely immersed. Let W equal the weight of the body, P the force required to completely immerse it, then the force necessary to resist the resultant vertical pressure $= W + P =$ weight of water displaced by the whole body.

$$\text{Hence, specific gravity} = \frac{W}{W + P}.$$

What we want, therefore, is to determine P . This we can do by attaching a heavy body called a *sinker* to the light body, and by weighing them both in water. If they weigh zero, the sinker's weight in water is the force P required; but if they tend to sink and are found to weigh B , then the sinker's weight in water is too great by this weight B .

If, therefore, A equals the apparent weight of the sinker in water,

$$A - B = P,$$

and the specific gravity of the body is $\frac{W}{W + A - B}$.

For determining these weights the hydrostatic balance is again employed.

§ 56. To find the specific gravity of a liquid, by weighing a solid in it. —

Let the solid weigh W in air, and let a be its apparent weight in the given liquid.

Then $W - a =$ weight of the volume of liquid displaced by the solid.

Let solid weigh A in water.

Then $W - A =$ weight of the volume of water displaced by the solid.

But these two volumes are the same;

$$\therefore \text{specific gravity of liquid} = \frac{W - a}{W - A}$$

§ 57. To find the specific gravity of a solid body soluble in water, but insoluble in a liquid of known specific gravity.

If the body is soluble in water, its weight in water cannot be directly found, and consequently the method of § 54 is inapplicable to this case. If, however, the body is insoluble in some other liquid the specific gravity of which is known, we have the means of determining the specific gravity of the body.

Suppose the weight of the body to be W , and its weight in the known liquid a , then $W - a$ is the weight of the liquid it displaces; and if s be the specific gravity of this liquid, $\frac{W - a}{s}$ = the volume of liquid occupied by the body. It follows, therefore, that the weight of an equal volume of water is $\frac{W - a}{s}$, since specific gravity of water is *unity*.

Hence, specific gravity of body = $W + \frac{W - a}{s} = \frac{Ws}{W - a}$.

§ 58. Specific Gravity of Gases.—In determining the relative weight of gases, air at 0°C , and at the ordinary atmospheric pressure is taken as the standard substance. The process is attended with so many difficulties, owing to the peculiar properties of gases which have not yet been considered in this work, that many precautions are needed in order to obtain accurate results. In comparing the weights of equal volumes of any gas and air, it is necessary that the gases should be subjected to the same atmospheric pressure, and should be brought to the same temperature.

If we suppose W to be the weight of a glass globe full of air, and w its weight when empty, and if W'

be the weight of the same globe filled with any other gas, then the specific gravity of this gas as compared with air is

$$\frac{W' - w}{W - w}.$$

If the globe be filled with water from which all air-bubbles have been carefully excluded, and if W_1 be its weight so filled, the specific gravity of the gas as compared with water would be $\frac{W' - w}{W_1 - w}$.

In determining with accuracy the specific gravity of any substance, the temperature at which the experiment is conducted must be considered; for it is known that bodies generally expand when heated, and the weight of the same volume of water, or of any other substance, consequently varies at different temperatures. This subject is more fully considered in treatises on heat.

Before working the following exercises the student is recommended to take pieces of brass, tin, zinc, marble, wood, and other easily obtained substances, and to determine their specific gravities by the hydrostatic balance, comparing the results with those given in the tables.

EXERCISES. VII.

1. A solid soluble in water but not in alcohol weighs 346 grams in air and 210 in alcohol. Find the specific gravity of the solid, that of alcohol being 0.85.

2. A solid weighs 100 grams in vacuo, 85 grams in water, and 88 grams in another fluid. What is the specific gravity of the fluid?

3. A piece of copper weighs 31 grains in air and 27·5 in water. Find its specific gravity.

4. A piece of wood (sp. gr. = 0·74) of 32 cubic inches floats in water. How much water will it displace?

5. A body floats in water with one-eighth of its volume above the surface: determine its specific gravity. How much of it will be submerged in a fluid whose specific gravity is 0·9?

6. A body floats in a liquid (sp. gr. = 13·5), and $\frac{2}{3}$ of its volume is above the surface: find specific gravity of the body.

7. If a ball of platinum weigh 21·4 oz. in air, 20·4 oz. in water, and 19·6 in hydric-sulphate, find the specific gravity of the platinum and of the acid.

8. A piece of marble, specific gravity = 2·84, weighs 92 grams in water and 98·5 grams in oil of turpentine. Find the specific gravity of the oil.

9. A piece of cork weighs 2 oz., and its specific gravity is 0·24. What is the least force that will just immerse it?

10. A piece of iron, sp. gr. 7·21, and weighing 360·5 grams is tied to a piece of wood weighing 300 grams, and the weight of both in water is 110·5 grams. Find the specific gravity of the wood.

11. A cubic block of wood each edge of which is 8 cm. is put into water. If the specific gravity of the wood is 0·85, with how much more wood must it be loaded, so that its upper surface may sink to the level of the water?

12. A cylinder of wood 20 inches in height, and a cylinder of lead 1 inch in height, are united so as to form one cylinder 21 inches in height, which is found to float in water with 3 inches projecting above the surface. Find the specific gravity of the wood, that of the lead being 11·4.

X. *Other Methods of obtaining the Specific Gravity of Substances.—Hydrometers.*

§ 59. **The Specific Gravity Bottle.**—This is a bottle made to hold a certain weight of water at the

standard temperature. It may be employed for determining the specific gravity of a liquid or a powder.

1. *Specific Gravity of a Liquid*.—Suppose the bottle when empty to weigh w grams, and that it holds 100 grams of water. Let the bottle be filled with the liquid the specific gravity of which is required, and suppose it then to weigh $(91 + w)$ grams. Then the weight of the liquid in the bottle is 91 grams, and the weight of an equal bulk of water is 100 grams.

Hence, specific gravity of liquid is $\frac{91}{100} = 0.91$.

2. *Specific Gravity of a Powder*.—Let the powder be first placed in the bottle, and let the bottle be then filled with water, and suppose the weight of the contents of the bottle, *i.e.*, of the powder and water, to be K . Then if W be the real weight of the powder, and Y the weight (unknown) of the water it displaces,

$$K = 100 + W - Y,$$

and

$$\therefore Y = W + 100 - K$$

$$\therefore \text{Specific gravity of powder} = \frac{W}{W + 100 - K}$$

Take 4 grams of powdered glass, put it into the bottle, and fill with water; then the bottle with its contents will be found to weigh $102.5 + w$ grams;

$$\therefore Y = 4 - 2.5 = 1.5; \therefore \text{specific gravity} = \frac{4}{1.5} = 2.6.$$

§ 60. **Hydrometers**.—The hydrometer¹ consists essentially of a straight stem, loaded at one end, so as

¹ This instrument is said to have been invented by Hypatia, the daughter of Theon Alexandrinus, who flourished about the end of the fourth century; though there is some foundation for the opinion that the invention is due to Archimedes.

to float in a vertical position. By observing the depths to which it sinks in two different liquids, the relative weights of these two liquids can be determined. Hydrometers are of very different forms, and may be used for finding the specific gravity of liquids or of solid bodies. We shall describe two varieties only.

§ 61. **Common Hydrometer.**—This instrument, invented by Fahrenheit,¹ consists of a straight stem, usually made of glass, which terminates in two hollow spheres. The lower sphere is loaded with mercury, so that the instrument may float in a vertical position.

FIG. 30.



Let a = section of the stem.

w = weight of the instrument.

v = volume „ „

Suppose the instrument to float in water with the point D of its stem on the surface of the water, and in some other liquid, the specific gravity of which is to be found, with the point C on the surface.

Then, if s be the specific gravity of the liquid, it follows from the principle of Archimedes that

$$w = s(v - a \times A C) = v - a \times A D$$

taking the weight of the unit-volume of water as unity.

$$\therefore s = \frac{v - a \times A D}{v - a \times A C}.$$

§ 62. **Nicholson's Hydrometer.**²—This form of instrument enables us to determine the specific gravity

¹ 1724.

² 1787.

of solid as well as of liquid bodies. It consists of a hollow body, B, connected by a fine stem with a small dish, A, at its upper end, and at its lower end with a cup, C, loaded so as to ensure equilibrium.

When floating in distilled water, with a certain weight, k , in the upper dish, the instrument sinks to such a depth that a fixed mark, o , is on the surface of the water.

1. If we want to find the specific gravity of a given liquid, we place the instrument in it, and place weights in the upper dish till the fixed mark, o , is on a level with the surface of the liquid.

If, then, W be the weight of the instrument, k the weight added to sink it to o in water, and w be the weight added to sink it to o in the given liquid, $W+k$ is the weight of the water displaced; and $W+w$ is the weight of the liquid displaced; and, as the same bulk of fluid is displaced in each case, the specific gravity of the liquid is

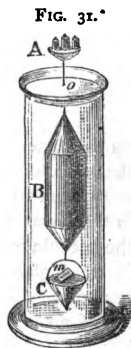
$$\frac{W+w}{W+k}.$$

2. To find the specific gravity of a solid.

Let k be the weight as before that must be placed on the instrument to bring the point o on a level with the surface of the water. Place a small piece of the solid on A, and diminish the weight k , so as to keep the instrument at the same level.

Let m be the weight now employed.

Then $k-m =$ weight of solid body.



Now place the substance the specific gravity of which is to be found in the cup *c*, in which case a weight greater than *m* by the weight of the water displaced by the body must be placed on *A* to maintain the instrument in the same position.

Let *n* equal the weight now in *A*.

Then $n - m$ = the resultant vertical pressure on the solid in *c*,

= the weight of the water displaced by it.

Hence, specific gravity is $\frac{k-m}{n-m}$.

EXERCISES. VIII.

1. A piece of glass the weight of which is 50 grains is placed on the upper dish of a Nicholson's hydrometer, and it is found that an additional weight of 235 grains is required to sink the instrument to the fixed level. If, however, the glass be placed in the lower cup a weight of 250 grains is required. Determine the specific gravity of the glass.

2. A globe of glass holds 5 litres of water at standard temperature. When full of water it weighs 5500 grams. When full of air it weighs 506.465 grams, and when full of hydrogen it weighs 500.447 grams. Find the specific gravity of air as compared with water, and of hydrogen as compared with air.

3. A bottle filled with water is found to weigh 500 grams. If 180 grams of powder are introduced into the bottle, the bottle with its contents weighs 575 grams. Required the specific gravity of the powder.

4. Into a specific-gravity bottle capable of holding 1000 grains of water, 300 grains of a certain powder are introduced. The bottle is then filled up with a liquid, specific gravity 0.8, and the contents of the bottle are found to weigh 980 grains. Find the specific gravity of the powder.

CHAPTER V.

THE MOTION OF LIQUIDS.

XI. *Liquids moving by their own weight.*

§ 63. It is proposed to consider in this chapter a few of the most elementary propositions connected with the movements of fluid bodies. Very little can be attempted without the application of some of the higher processes of mathematics, and we shall endeavour, therefore, only to indicate the principles of the methods on which the solution of this class of problems depends. As all liquids are more or less viscous, absolute agreement cannot be expected to exist between the conclusions arrived at on the assumption of a frictionless fluid and the results of direct experiments.

§ 64. **Definition.**—The vertical distance between the surface level of a liquid in a vessel and the centre of the orifice through which it escapes is called the *head* or *pressure height* under which the flow takes place.

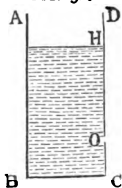
§ 65. **Torricelli's Theorem.**—The velocity with which a liquid escapes from a small hole in the bottom or side of a vessel was the subject of numerous experiments made by Torricelli.¹ The result at which he

¹ Born in Italy in 1608 ; died in 1647.

arrived was that the rate of efflux was the same as would be acquired by a body falling freely from the surface level of the liquid to the centre of the orifice from which it escapes.

Let A B C D be a vessel having a small opening, O, in one of its sides. Let the area of this opening,

FIG. 32.



ing, which is supposed to be very small, be a , and suppose a small mass of liquid, m , the thickness of which is c , to occupy this opening.

Then if p be the pressure in excess of the atmospheric pressure urging this mass forwards, measured in units of force per unit-area, $p = g \cdot d \cdot a \cdot h$ where d is the density of the liquid, and h the pressure height, O H ; and the work done by this force p in urging the mass m from a position of rest through the distance c is

$$p \times c = g \cdot d \cdot a \cdot h \cdot c ;$$

and since the work done is equal to the energy which the mass m acquires,

$$p \cdot c = \frac{m v^2}{2}, \text{ where } v \text{ is velocity of efflux ;}$$

$$\therefore g \cdot d \cdot a \cdot h \cdot c = d \cdot a \cdot c \cdot \frac{v^2}{2}$$

$$\text{or} \quad v^2 = 2 g h$$

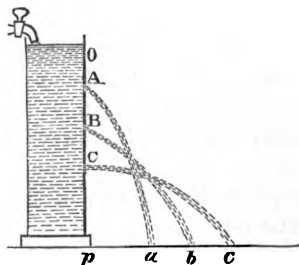
i.e., the velocity is that which would be acquired by the mass m in falling freely from the surface level of the liquid to the centre of the orifice.

In this result each particle of the mass is supposed to move at right angles to the area of the orifice, and to escape into the air at a pressure equal to that at the surface level.

§ 66. **Experimental Test.**—The accuracy of the results given by Torricelli's theorem may be tested

for different liquids, by taking a vessel having apertures in one of its sides through any of which the water or other liquid it contains may escape. If the vessel be kept constantly full, and one of these apertures be opened, the liquid will flow through

FIG. 33.



it, and will descend in a curve similar to the path of a body projected horizontally from a certain height above the ground.

If the distances p , a , b , c be observed, the accuracy of Torricelli's theorem can be tested.

For, let the height $0c = h$, then according to the theorem, if v be the velocity of the liquid at c , $v = \sqrt{2gh}$, and the velocity will be uniform and horizontal.

If then t equal the time occupied by a particle in moving from c to c , we know by the second law of motion that t equals time occupied by a body falling freely from c to p .

= time of describing pc with the uniform velocity v .

Hence, $p c = t v$; also $c p = \frac{t^2 \cdot g}{2}$

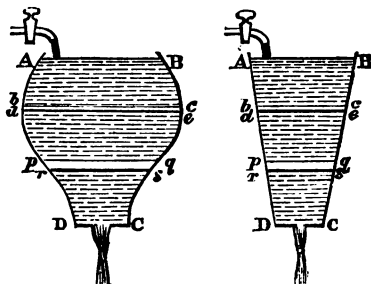
$$\therefore c p = \frac{(p c)^2 g}{2 v^2},$$

$$\text{or } v = p c \times \sqrt{\frac{g}{2 c p}}.$$

Now, if the distance $p c$ be known, and if $p c$ be observed, the value of v so found can be compared with that given by the formula $v = \sqrt{(2 g \cdot o c)}$. On comparing the results, obtained by experiment, with those obtained from Torricelli's theorem, a certain difference will be always found to exist. Some of the reasons of this difference, apart from those dependent on the viscosity of the liquid, will be considered later on.

§ 67. Relation of Velocity of Flow to Sectional Area of Vessel.—Let $A B C D$ be a vessel, which is

FIG. 34.



kept constantly full of liquid. We may consider the mass of the fluid, which is supposed to be incompressible, to be divided into small laminæ moving

parallel to one another and of uniform sectional area for very small differences of depth. Thus, if $b c d e$, $p q r s$, be two small laminæ, we may suppose $b c = d e$ and $p q = r s$. Let each particle in the section $b c$ have the velocity, v , and each particle in the section $p q$ the velocity v' . If, then, the section $b c$ descend to $d e$ in one second, the section $p q$ will descend to some depth, $r s$, in the same time ; and the volume $b c d e$ will be equal to the volume $p q r s$, since the quantity of liquid between $d e$ and $p q$ remains constant.

If, then, A equal the section $b c$ or $d e$, and A' the section $p q$ or $r s$,

$$A v = A' v' = \text{volume discharged in one second,}$$

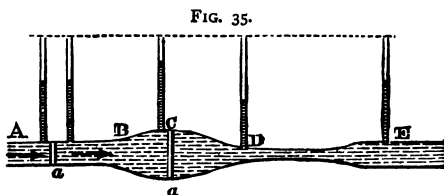
$$\therefore v : v' :: A' : A,$$

or the velocities are inversely proportional to the sectional areas.

§ 68. **Vena Contracta.**—When a liquid issues into the air from a small opening in a thin plate, the stream is found, first of all, to converge, so that it contracts rapidly for some little distance from the orifice. This fact may be very easily observed, and will be seen in all cases such as those shown in figs. 33, 34. The area of the jet at its narrowest part is known as the *vena contracta*, and was first made the subject of investigation by Sir Isaac Newton. It is found to be generally about three-fifths of the aperture of the orifice, but varies with the shape of the aperture and the pressure height. The value of this fraction, which is called the *coefficient of contraction*, can be determined by experiments only. In all calculations with respect to the rate of the emptying of vessels, the method

usually adopted has been to apply the results of Torricelli's theorem, and to substitute the area of the vena contracta for that of the orifice. This method, we shall see, gives us a merely empirical result, and is faulty in so far as it overlooks some important elements in the problem.

§ 69. **Measure of Pressure on the Walls of a Pipe.**—Suppose a liquid is flowing through a pipe A B, the pressure which it exerts on the walls of the pipe at any point can be experimentally determined by inserting gauge glasses, *i.e.*, thin glass tubes of about



$\frac{3}{8}$ of an inch diameter, into the pipe at those points at which the pressure is to be found. Since fluids transmit their pressure equally in all directions, the forward pressure exerted by the moving liquid will be equally exerted on the walls of the pipe, and will force the liquid up the gauge glasses to a height corresponding to the pressure produced. If the pipe is of uniform sectional area the pressure produced at all points will be the same, friction being neglected, and the liquid in the pipe will rise in the gauge glasses to the same height at all points in the pipe.

§ 70. **Relation of Pressure to Sectional Area of Pipe.**—If the sectional area of the pipe varies, the

pressure is no longer uniform, but is found to be greatest where the area is greatest, and *vice versa*.

Suppose the liquid to be moving with a uniform velocity in that part of the pipe A B (fig. 35) which has a uniform sectional area. If, then, we consider an element a of the liquid, we see that the pressure on either side of a must be the same, since any increase of pressure from behind would cause it to move with an acceleration, which is not supposed to be the case.

Suppose now the element to have reached B, and to be entering the wider part of the pipe, its velocity in the direction of the pipe's length will now be less than it was before, and will continue to decrease as the pipe widens. Hence the pressure of the liquid behind the element a urging it on must ever be less than that in front of it resisting its advance, and consequently the pressure must increase with the area of the pipe and with the decrease in the velocity of motion.

After passing C, the widest part of the pipe, the velocity begins again to increase, and as the resistance to the forward motion of the particles must consequently have become less, the pressure is also diminished.

These results can be practically illustrated by observing the height of the liquid in the several gauge glasses placed at different points in the pipe. Due allowance must be made for the frictional resistance which causes the liquid to rise to a less height in the glasses than it otherwise would, and gives a difference between the theoretical and actual results which in-

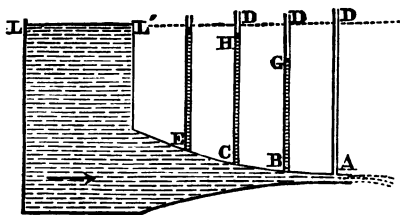
creases with the length of pipe traversed. This resistance being neglected, we see that the pressure which a liquid exerts on the sides of a pipe through which it is passing varies directly with the sectional area, the velocity at the two ends of the pipe being the same.

If p_1 and p_2 be the pressure exerted at points where the area of the pipe is a_1 and a_2 , we have

$$p_1 : p_2 :: a_1 : a_2.$$

§ 71. Relation of Velocity of Flow to the Pressure produced.—We have seen that in the case of a steady flow through a pipe of varying area, the velocity at different points varies inversely with the area, friction being neglected. We have now to consider how the pressure changes with the velocity. This may be experimentally exhibited by fixing the larger end of a tapering pipe into a vessel of water kept constantly full, the upper surface having only the ordinary atmospheric pressure upon it. If the pipe be furnished

FIG. 36.



with gauge glasses, the pressure at different points can be observed. The hydrostatic pressure at A is evidently zero, or the same as that of the air into

which it issues, but the liquid will be found to stand at a continually higher level in the gauge glasses as the pipe widens towards the vessel. The difference between the surface level of the water in any one of the gauge glasses and that of the vessel is called the *fall of free level* at that point, and this fall is equal to the difference between the height of the column of liquid representing the hydrostatic pressure at a point in the stream, and the pressure-height at the same point, supposing the liquid to be in equilibrium.

Consider now a small mass m of the liquid in the section of the stream at any point B, and let the section of this mass be equal to a unit of area, and its length equal to c , so that it contains c units of volume. Then if p be the actual pressure in units of force per unit of area at B, and if P be the pressure at the same point due to the depth of the mass m below the free surface $LL'D$ of the liquid, then the whole amount of work which the mass m has received from the pressure behind it in moving through its own length from a position of rest is $P \times c$, and since p is the pressure it is exerting, the work which it gives to the liquid in front of it is $p \times c$. Hence the excess of work which it receives is $(P-p)c$.

Now if H is the vertical depth of m below LL' , and h the height of the liquid in the gauge glass above m ,

$$P = g H d \text{ and } p = g d h,$$

where d is the density of the liquid,

$$\therefore (P-p)c = g d (H-h)c.$$

But the amount of work gained by the mass m is equal to its energy.

Hence, $g d (H-h) c = \frac{m v^2}{2}$, where v is the velocity of mass m .

And since $m = d c$, we have

$$v^2 = 2 g (H-h) = 2 g D G$$

or, the velocity at any point in the stream is that due to the difference between the statical and actual pressure-height of the moving particles. Hence, *in steady flow the velocity generated from rest is that due to the fall of free level.*

If we suppose the sectional area at B to be twice that at A, and at C twice that at B, and so on, then if v be the velocity at A, the velocity at B = $\frac{v}{2}$, since the velocity varies inversely with the area in cases of steady flow ;

and since $v^2 = 2 g \cdot A D$,

we have $\frac{v^2}{4} = 2 g \cdot D G$, or $B G = \frac{3}{4} A D$;

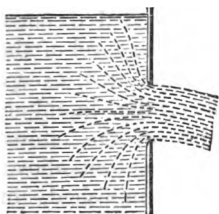
and $\frac{v^2}{16} = 2 g \cdot D H$, or $C H = \frac{15}{16} A D$, and so

on, which gives the relation between the actual hydrostatic pressure at a point, and the velocity with which the stream is passing that point.

§ 72. Application to the Flow of a Liquid through an Orifice in the Base or Side of a Vessel.—When a liquid flows through an orifice, as previously considered in the case of Torricelli's theorem, the flow does not actually take place in a direction at right

angles to the orifice, at all points of the orifice, but in directions such as are shown in fig. 37. The

FIG. 37.



nature of the flow is therefore somewhat similar to that indicated in the preceding section, in which the varying area of the pipe is represented by the space included between the several stream lines. At all points along the margin of the orifice the direction of the motion is tangential to the

plane of the orifice, and the water comes at once into contact with the atmosphere. But at all intermediate points the issuing stream exerts a pressure greater than that due to the atmosphere, and consequently the velocity at any of these points is not that due to the height of the free surface of the liquid in the vessel, but is that due to the difference between this height and that of the column of liquid corresponding to the actual pressure at the particular point in the moving stream. This difference of surface level has already (§ 71) been referred to as the fall of free level; hence the velocity of any particle in the issuing stream is that due to the fall of the free level; or, if h equals the depth of a particle in the stream below the free surface of the liquid, and z equals the height of the column of liquid representing the pressure at that particular point, then if v be the velocity with which this particle is moving,

$$v = \sqrt{2g(h-z)}.$$

In calculating the actual discharge in a given time

through a small orifice in the base or side of a vessel, the following principles, which are essential conditions of the flow, have to be considered :

1. The absolute velocity at any point in the plane of the orifice is not that due to the vertical distance between the point and the free surface level of the liquid, since everywhere throughout the area, except along the boundary of the orifice, the liquid is under a pressure greater than that of the atmosphere, and consequently, as we have shown above, the velocity is correspondingly diminished, or $v = \sqrt{2g(h-z)}$.

2. If the orifice be divided into small horizontal bands, we cannot suppose the velocity to be constant throughout each band, since the direction of the motion of the fluid is different at different parts.

3. If we consider any vertical section of the liquid as shown in the figure, since the direction of the motion towards the orifice is nowhere perpendicular to the orifice, except at its centre, the actual velocity of efflux is only the component of the whole velocity as determined by the first of these principles, acting in a direction at right angles to the plane of the orifice.

These considerations show how very complicated is the problem of determining the actual discharge of a liquid, per unit of time, through a given orifice, and serve to indicate some of the causes of the discrepancies between the results of actual experiments and those given by Torricelli's theorem. Ordinarily, when the orifice is small, the discharge is roughly calculated by considering the velocity of efflux to be that due to the distance between the centre of the orifice and the

level of the free surface, and by then substituting the area of the *vena contracta* for that of the orifice. Thus if a be area of a very small orifice, and γ the coefficient of contraction, and if h be the vertical distance of the centre of the orifice from the free surface level, the discharge per unit of time is taken as $\gamma a \sqrt{2gh}$. But this result is not only a mere empirical result, the degree of accuracy of which depends on the value of γ as experimentally determined, but it leaves out of consideration some important circumstances under which the flow actually takes place.¹

§ 73. Effect of Friction on the Pressure of a Liquid Flowing Uniformly through a Pipe.—Hitherto we have taken no account of the frictional resistance which retards the passage of a liquid through a pipe, and which, apart from all other circumstances, causes the velocity of efflux to be less than that due to the entire pressure-height above the orifice from which the liquid escapes into the air.

The effect of this frictional resistance on the pressure exerted by a liquid in flowing through a pipe of uniform sectional area may be conveniently illustrated by the following arrangement.

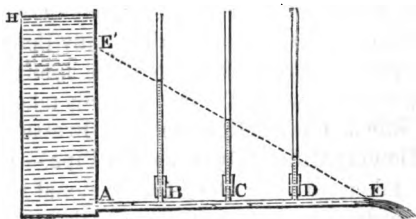
HA (fig. 38) is the section of a cylindrical zinc vessel about 3 ft. high, into the side of which, near the bottom, is fixed a brass pipe of uniform bore. At equal distances along this pipe are openings into which glass gauge-glasses are fitted.

If the opening E be closed, and the apparatus filled with water, the liquid will stand at the same

¹ 'On the Flow of Water through Orifices.' By Prof. James Thomson. Report of British Association, Glasgow, 1876.

level in the vessel and in the pressure-tubes ; but as soon as the end *E* of the tube is opened, the water falls to different levels in the several tubes, being lowest in the tube nearest the end, and rising by equal differences in the several tubes. If the water in the vessel be kept at the same level, it will be observed

FIG. 38.



that the surface levels of the liquid in the several tubes lie on a straight line drawn through *E* to some point *E'*. This line is found to be more nearly horizontal, the greater the sectional area of the pipe through which the water flows.

Now it is evident that if the liquid during its flow had encountered no frictional resistance, its velocity on leaving the vessel *A H* would have been that due to the pressure-height above it, and its pressure on the tube would not have exceeded that of the atmosphere into which it issued. The additional pressure on the tube is due, therefore, to the frictional resistance encountered by the liquid ; and as the resistance to be overcome at any point of the tube increases with the distance of that point from the orifice, the pressure of the liquid diminishes as it approaches the open end.

Let a equal the sectional area of the pipe, and let c be

the length of an element of liquid occupying it. Let f be the measure of the frictional resistance per unit of length of pipe, and let p and p' be the actual pressures per unit area at either end of the element c . Then if we suppose m to be the mass of this element, and v its velocity, which is constant, $\frac{m}{2} v^2$ represents the energy of this mass at all points in the pipe.

Now if the mass m moves through a distance c , the work which it has received is $p a \times c$, and the work which it has given to the liquid in front of it is $p' a \times c$. Hence the resultant work gained in moving through the space c is $(p a - p' a) c$. But since the energy of the mass remains constant, this amount of work is employed in overcoming the frictional resistance encountered by the liquid in its passage through the pipe.

The work done against friction is $f \times c$; and

$$\therefore (p - p') a c = f \times c;$$

and since $p - p' = g d (h - h')$

where h, h' are the heights of water required to produce the pressures p and p' respectively, and d is density of the liquid, we have

$$h - h' = \frac{f}{g d a},$$

which shows that the difference of pressures (as measured by height of the liquid in pressure-tubes) for equal distances along a pipe of uniform sectional area is constant.

It is to be here observed that whilst the kinetic energy of the flowing water is constant, there is a continual decrease of potential energy much in the same way as when a body slides with a uniform velocity down a rough inclined plane.

XII. *Capillarity.*

§ 74. In the preceding sections we have treated of the equilibrium and motion of liquids in mass, but we shall now consider a number of phenomena which are mainly due to the action of forces on the molecules of a liquid, and which are not immediately explicable by the principles already stated. These will be discussed under the two heads of Capillarity and Diffusion, and they comprise the formation of drops, the relation of liquids and solids in contact, and the intermixture of liquids, either in contact with one another, or separated by porous membranes. In discussing these subjects we shall in each case commence with a few experiments, and then indicate the nature of the principles which seem to explain them.

§ 75. **Drop Formation.—Experiments.**—Take a vessel containing olive oil and carefully drop into it some small portions of water. These will be seen to assume a spherical shape as they descend slowly through the oil. Raindrops are spherical; but the rapidity with which they fall through the air is so great that we are unable to see them long enough to distinguish their form.

To show that a large amount of liquid will assume a spherical shape if left free to the action of its internal forces, mix together alcohol and water, in such proportions that the mixture may have the density of olive oil. This will require about three-parts of alcohol to one of water. Then pour gently, so that it may not be broken into parts, some olive oil into the

mixture, and it will be found to float in the form of a sphere in any part of the surrounding liquid. Without any great difficulty globules of oil may in this way be obtained, having a diameter of four or five inches. These can be flattened between two glass plates, or receive an indentation by being touched with a glass rod, without losing their cohesion ; and as soon as the external body is removed, they recover like an elastic ball, their spherical form.

If pure quicksilver be scattered on a level surface of glass, the smallest of the drops will differ very little in form from perfect spheres. The larger drops, owing to their weight, will be considerably flattened ; and if a greater quantity cohere together, the tendency to a spherical shape is visible, only in the somewhat curved form of the upper surface.

In blowing an ordinary soap-bubble we obtain a very good idea of the nature of the forces that act on a fluid surface. For this purpose 'take some common soapsuds, or a Plateau's mixture of soap and glycerine, and blow a small bubble at the end of a tube with a bell mouth. A tobacco-pipe will answer if the bore of the tube is large enough. After blowing the bubble at one end of the tube, place the other end near the flame of a candle. The bubble will contract and drive a current of air through the tube, as may be seen by its effect on the flame.

'This shows that the bubble presses on the air within it, and is like an elastic bag. To enlarge the bubble by blowing into the tube, work must be done to force the air in, because the pressure inside the bubble is greater than that of the air outside. This

work is stored up in the film of the soapsud, for it is able to blow the air out again with equal force.'

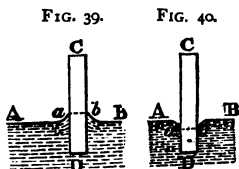
§ 76. **Surface-Tension.**—These experiments show that when two fluids are in contact with each other and do not mix, the surface separating them is in a state of *tension* similar to that of a membrane stretched equally in all directions, and that the surface-particles have a tendency to approach one another like those of the outer side of a bent watch-spring. This contractile force, or surface-tension, resides in the thin film separating the two fluids, and does not extend to any appreciable depth below the surface; and to its action is due the spherical form of the drop and of the soap-bubble. In the case of the soap-bubble the superficial tension may be measured by considering the work done in order to produce a film of a certain area; and the measure of the tension per unit length is found to be the same as the numerical value of the work done divided by the area.¹

This tension seems to be due to the fact that whilst the particles in the interior of a fluid are subjected to equal forces on all sides, those on the surface separating the two fluids are under the influence of different molecular attractions, the result of which is to give to the bounding surface a tendency to contract. The surface-tension depends on the nature and temperature of both media, decreasing as the temperature rises, and vanishing altogether at that temperature at which there is no longer a well-marked distinction between the liquid and gaseous states.² Between any two liquids that do not mix,

¹ C. Maxwell, *Theory of Heat*, pp. 281, 283. ² § 4.

between a liquid and its own or another vapour, and between a solid and a fluid of any kind, there is a definite surface-tension, the value of which for a unit of length is called the co-efficient of superficial tension, or co-efficient of capillarity, as having been first considered in connection with the ascent of liquids in capillary tubes. This tension may be reckoned in C. G. S. units of force per linear centimetre, and it is found that at a temperature of 20°C . the tension of a surface of water in contact with air is 81, that of mercury and air 540, that of alcohol 25.5, and of olive oil 36.9 units of force. In fact, of ordinary liquids water is found to have the greatest surface-tension.

§ 77. **Capillary Elevation and Depression.**—**Experiments.**—(1.) If a sheet of clean glass be immersed in water and then removed, the water will be found to have wet the surface of the glass. If, whilst partially immersed, the surface of the water in contact with the glass be observed, it will be found that the surface of the water in the immediate neighbourhood of the sheet of glass is raised. If we perform the same experiment with mercury instead of water, we



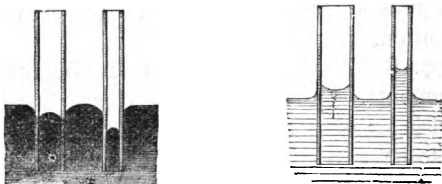
shall find that the mercury does *not* wet the glass, and that the surface of the mercury in the neighbourhood of the glass will be *depressed*.

If A B be the surface of the liquid, c D the vertical section of the plate of glass, the water (fig. 39) will be found to rise to some

height $a b$, and the mercury (fig. 40) to be depressed to the level $a' b'$. Moreover, the surface of the water in contact with the glass will be *concave*, and that of the mercury *convex*; and the elevation of the water will be found to be somewhat greater than the depression of the mercury.

(2.) Take now two plates of glass and let them approach each other in water. When they are sufficiently close, the water will begin to *rise* between them,

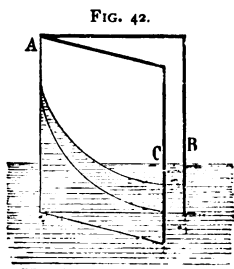
FIG. 41.



and the height to which it rises will increase as the distance between them lessens. Moreover, the surface of water between the plates will be *concave*. If the same experiment be made with mercury instead of water, the mercury will be *depressed* between the plates, but to a *less* extent, and the surface will be *convex*.

If this experiment be varied by taking plates of different thickness, the same results will be obtained, thereby showing that whilst the amount of ascent or depression depends on the distance between the plates, it is independent of their thickness.

(3.) If instead of plain surfaces parallel to one another we take two plates of glass inclined at an angle, and immerse them partly in water, the water will rise between the plates to different heights, and the intersection of the surface of the water with the plate will form a curve, which is known as the rectangular hyperbola.



(4.) If glass tubes of small bore, called capillary tubes (from *capilla*, a hair), be partially immersed in water, the liquid will rise in the tubes, and the height to which it reaches will be found to vary inversely with the diameter of the tube. This result holds good whether the experiment be made in air or in a vacuum, but the elevation is found to diminish as the temperature of the water increases. If mercury be used instead of water, the liquid will be depressed, as previously stated in the case of parallel plates. The curved surface of the liquid is called a *meniscus*, and the meniscus is concave for water and convex for mercury.

§ 78. **Principles of Capillarity.**—When a liquid comes in contact with a solid, the particles of the liquid on the common surface of the two substances are to some extent in a similar condition to those of the free surface of a liquid; *i.e.*, they are not equally attracted on all sides, as is the case with particles in the interior. The interaction of the molecular forces of the surface of the liquid and solid

gives rise to a tension in the common surface that separates them. When a solid is in contact with two fluids there is a different tension in each of the surfaces separating a pair of media. In this case, in order that the three tensions acting at any point in the line of intersection of the surfaces may be in equilibrium, their directions must be such that they can be represented by the sides of a triangle, and any one must be greater than the difference between the other two. Now we have seen that the surface of a liquid in contact with a solid is curved, and for any given liquid in contact with a given solid there is a definite angle of contact, called the *angle of capillarity*, which is *acute* in the case of mercury, and *obtuse* in the case of water in contact with glass. This inclination of the surface of a liquid to that of a solid at the margin of contact is due to the excess of the tension of the surface separating the two fluids (for example, air and water, or air and mercury) above the difference of the tensions of the surfaces separating the solid from each of the fluids.

§ 79. Height of Elevation or Depression of Liquid in Capillary Tubes.—Let T denote the magnitude of the superficial tension per unit length of the free surface of the liquid inclined at a given angle to the solid in contact with it. Then the vertical component of T acting upwards or downwards is the force tending to raise or depress the liquid. Let t be the vertical component of T , and let r be the radius of the tube, then $2\pi rt$ denotes the magnitude of the vertical force acting all round the tube. If, now, s be the specific gravity of the liquid, h the mean height to which it is raised or depressed, then

we have $\pi r^2 h s$ equal to the weight of liquid supported,
and

$$2 \pi r t = \pi r^2 h s,$$

$$\text{or } h = \frac{2 t}{r s}$$

which gives the *law of diameters*, viz., that the height of elevation or depression varies inversely with the radius of the tube.

Since the surface-tension acts uniformly throughout the film, it produces a resultant pressure normal to the surface, which is always directed towards the centre of curvature of the surface.

It follows, therefore, that in capillary elevation the liquid surface must be concave, and in capillary depression convex.

In the case of a narrow tube partly immersed in a liquid which is in contact with air on both sides of the tube, we see that when the surface of the liquid within the tube is concave, the pressure immediately below the surface is less than the atmospheric pressure by a force due to the concavity of the surface, and that the pressure goes on increasing till at a depth corresponding to the mean level of the external liquid it is equal to the atmospheric pressure. If the surface is convex, the pressure immediately below the surface of the liquid in the tube is greater than the atmospheric pressure by a force corresponding to the difference of the mean level of the liquid inside and outside of the tube. Thus, in the case of a concave meniscus the resultant normal force due to the surface-tension acts in a direction opposite to gravity, and so lessens its effect, and in the convex meniscus it increases it.

XIII. *Diffusion of Liquids.*

§ 80. **Diffusion.**—If two liquids of different densities, and susceptible of permanent admixture, be placed in contact with each other, they gradually become intermixed. The process by which these liquids combine is known as *diffusion*.

§ 81. **Graham's Experiments.**—The phenomena of diffusion were first carefully investigated by Professor Graham, and the results of his early experiments were published in 1850. In conducting these experiments Graham employed a number of small phials, each holding about 114 cc. of water. The necks were carefully ground and of a uniform aperture of about 3·15 c. in diameter. Into these phials he poured solutions of various salts, so that the liquid rose as far as the shoulder of each phial. The phials were then very carefully filled with cold water. Thus charged, each phial was closed by a glass plate and then placed in a glass jar, containing sufficient water to cover the mouth of the phial and to rise at least 2·5 c. above it. The plate was then

FIG. 43.



carefully removed and the whole apparatus maintained for a certain period of time at a fixed temperature. The phial was then withdrawn and the water in the jar having been evaporated, the amount of salt that had passed from the phial into the jar by this process of diffusion was determined by weight.

A very simple experiment showing the diffusion

of liquids is the following:—Take a tall cylindrical jar and fill it to about two-thirds of its height with a solution of litmus. Then by means of a funnel pour in very carefully some hydric-sulphate, so as to occupy the lower part of the jar. If the jar be set aside, the acid will be found, after the lapse of two or three days, to have diffused into the litmus solution, as shown by the consequent red colour it will have acquired.



FIG. 44.

§ 82. **Results of Graham's Experiments.**—By varying the experiments with respect to the nature of the salt, the density and temperature of the solution, and the time occupied in diffusion, Graham arrived at the following results:

1. The increase in density in the diffusion product corresponds very nearly with the proportion of salt in the solution.

Taking four different solutions containing one, two, three, and four parts of common salt to 100 parts of water, it was found that the quantities diffused at the end of equal times were very nearly in the proportion of 1 : 2 : 3 : 4, the variation from this result not exceeding one per cent.

2. The quantity of salt diffused in equal times increases with the temperature.

From an experiment with a 4 per cent. solution, it was found that with a rise of temperature of $15^{\circ}\text{C}.$, the quantity of salt diffused increased somewhat more than one-third.

3. The rate of diffusion for weak solutions of the same salt is nearly uniform, but varies considerably with the nature of the substance diffused.

Of the several substances employed in these experiments hydric-chloride was found to be the most diffusible, and albumen one of the least diffusible. Intermediate between these was common salt. Comparing common salt with sugar-candy and albumen, it was found that with solution of 20 parts of the solid substance in 100 parts of water, exposed for eight days at a temperature of 16° C., the specific gravity of the diffused solutions were respectively 1.126 ; 1.070 ; and 1.053.

The following table shows the approximate times of diffusion of equal weights of different substances, taking that of hydric-chloride as unity :

Hydric-Chloride.	1
Sodic-Chloride	2.33
Cane Sugar.	7
Magnësic Sulphate.	7
Albumen	49
Caramel	98

4. If a solution be taken of two different salts which do not chemically combine, it is found that each follows its own rate of diffusion. Thus the inequality of diffusion of two different salts supplies a method for their separation, to a certain extent, from each other. If a mixed solution of two corresponding salts of potash and soda be placed in the phial, the potash salt being more diffusive than the soda salt

will escape into the water outside, whilst the soda salt will be relatively concentrated in the phial.

5. It is also found, in the case of very weak solutions, that a salt will diffuse into water which already contains some other salt in solution, showing that the diffusion of one salt is not sensibly resisted by the presence of another.

§ 83. **Crystalloids and Colloids.**—By considering the different diffusibility of different substances, Graham found that all bodies might be referred to two classes, which he called *crystalloids* and *colloids*. The properties of these two classes of substances are very different. Bodies susceptible of crystallisation belong to the former class. They are highly sapid ; they generally form a solution which is very slightly viscous, and they diffuse rapidly through water, or through a porous diaphragm. Colloids, on the other hand (so-called from *κόλλη*, glue), to which class belong starch, gum, hydrated alumina, albumen, gelatine, and many organic compounds, are insipid and have very feeble chemical relations. They diffuse very slowly in water ; but they form a medium, like water, which arrests the passage of other colloids, but through which crystalloid substances are capable of diffusing. A peculiar property of these substances is their mutability. They pass very easily from the liquid to the curdled condition. They are often largely soluble in water, but are held in solution by a very feeble force. They never crystallise, and are generally distinguished by the sluggishness of the molecules composing them.

§ 84. **Dialysis.**—Many insoluble colloids are per-

meable, as water is, to highly diffusive substances, but effectually resist the passage through them of other colloid substances which may be in solution. If, therefore, a solution containing a crystalloid and colloid substance be placed in a vessel containing water from which it is separated by a septum, or membrane, formed of some insoluble colloid, the crystalloid will pass through the septum, and the other less diffusive substance will remain behind. This process of separation by diffusion through a colloid septum is termed *dialysis*.

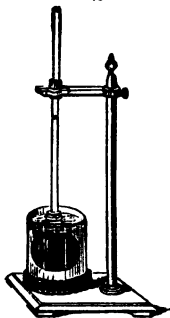
Experiment.—Take a sheet of very thin and well-sized paper having no porosity (so that if wetted on one side the other side remains dry) ; let the paper be thoroughly wetted and then laid on the surface of some water contained in a small basin of less diameter than the width of the paper. Having made a small depression in the paper, so as to form a cavity, place within it a mixed solution of cane sugar and gum arabic containing about 5 per cent. of each. In the course of twenty-four hours the water below will be found to contain about three-fourths of the sugar, which will have passed through the paper, leaving the gum behind.

What takes place during the process of dialysis may be thus explained :—A soluble crystalloid is capable of separating the water from the colloid, with which it is feebly united in the septum. It thus obtains a liquid medium for diffusion. The soluble colloid, on the other hand, is unable to separate the liquid from the colloidal substance of the septum, with which it is chemically combined, and is thus unable to find a way for its own passage by diffusion.

The process of dialysis is extensively employed for separating poisonous crystalloids from organic substances with which they may be mixed; and thus separated they yield more easily to the methods of chemical analysis.

§ 85. **Osmose.**—Intimately connected with the phenomena of diffusion is the interchange of liquids through porous diaphragms, which is known as *osmose*. Dutrochet¹ was the first to give his attention to this subject. The *endosmometer*, an instrument invented by him, illustrates this process. It consists of a long tube, connected at its lower end with a membranous bag which forms a reservoir. This is filled with a solution of sugar or gum, and is kept in water. After a time the liquid rises in the tube, and the level of the water falls in which the endosmometer is placed. Moreover, traces of the substance contained in the bag are found in the water outside. This shows that an interchange of the liquids has taken place, and that more liquid has passed inwards through the membrane than has escaped outwards from the reservoir. To this flowing-in of the liquid Dutrochet gave the name of *endosmosis*, and he called the reverse process, by which the liquid passes out from the membranous bag, *exosmosis*.

FIG. 45.



From numerous experiments Graham was led to

¹ *Nouvelles Recherches sur l'endosmose et l'exosmose*, 1828.

infer that in order to induce osmotic action between two liquids they must each be capable of acting chemically, but in different degrees, on the diaphragm separating them. Where porous materials are used not susceptible of chemical decomposition the osmotic action is very slight. When either of the liquids, by means of capillarity or by its action on the septum, or possibly by both processes, has effected a passage through the diaphragm, diffusion takes place, and the liquid rises in the tube. No perfectly satisfactory explanation, however, of these opposing currents of fluid has as yet been given.

The absorption of liquids by the spongioles of plants, and the interchange of liquids, which is constantly taking place in the animal body, in the processes of nutrition and secretion, are probably due to osmotic action and liquid diffusion.

§ 86. **Molecular Structure of Liquids.**—The phenomena of diffusion are interesting as affording us some insight into the molecular structure of liquid bodies. For, seeing that the molecules of one liquid are readily displaced by the molecules of another liquid into which they are diffused, it is evident that these molecules must be in a state of constant agitation and must be capable of continually changing their positions relatively to one another. In this way, therefore, the interchange of the molecules of two liquids in contact with each other can be explained; and the degree of diffusibility of any soluble substance would depend on the extent of the excursion which the molecules of the solution undergo, or on the dis-

tance through which a molecule can travel before it has its direction changed by impact upon another.

Thus the sluggishness of a liquid colloid would be accounted for by the very small range of the motion of its molecules ; and this explanation is supported by the fact already referred to, that these liquids are frequently highly viscous, and pass very readily into the solid state—a condition of matter in which the particles, though probably capable of revolving about an axis and of oscillating within certain limits about their mean positions, are not supposed to undergo any motion of translation whatever.

CHAPTER VI.

THE PRINCIPLES OF PNEUMATICS.

XIV. *General Properties of Gases.—Atmospheric Pressure.*

§ 87. **Pneumatics.**—The application of the principles of dynamics to the investigation of the phenomena presented by gaseous bodies constitutes a branch of Hydrodynamics known as Pneumatics.

§ 88. **Expansibility and Compressibility of Gases. Experiments.**—The characteristic properties of gaseous as distinguished from liquid bodies are shown by the following experiments :—

1. If a small quantity of gas be admitted into an empty vessel it will immediately expand, so as to occupy the whole of it.

2. If a bladder containing gas be placed under the receiver of an air-pump, from which the air is gradually removed, the gas contained in the skin is found to expand with the diminution of the external pressure.

3. If a cylindrical vessel fitted with a piston contain a certain quantity of gas, and the piston be pressed downwards by a weight or some other force,

the volume of the gas will be diminished ; and if the pressing force be removed the gas will expand.

It thus appears that the volume, which a certain quantity of gas occupies, depends on the pressure to which it is subjected, and changes with the size of the vessel containing it.

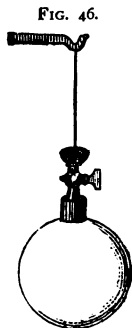
§ 89. **The Air.**—The earth is surrounded by a gaseous envelope reaching to a considerable height above the earth's surface. It consists of a mixture of two gases, nitrogen and oxygen, with small and variable proportions of other gases, especially of carbonic-dioxide. By the experiments of MM. Pictet and Cailletet, previously referred to (§ 4), air and its two principal constituents have been separately reduced to the liquid state. Carbonic-dioxide has long since been known to be a liquefiable gas.

As the majority of experiments connected with gases must necessarily be performed in air, it is very desirable, at starting, to consider some of its properties, as well as the various effects due to its action.

§ 90. **The Air has Weight.**—That the air is a heavy fluid was not known till comparatively modern times. Its invisibility, and its relative position with respect to all ponderables underlying it, may have helped to conceal this important fact from the knowledge of early observers. It appears that Aristotle,¹ suspecting the truth of this fact, and wishing to verify his belief, weighed a skin first empty and then inflated with air ; and finding it to weigh the same in both cases, concluded that air was a weightless fluid. The failure of this experiment was due to his having overlooked

¹ B.C. 384-322.

the fact that the skin when inflated with air occupied a correspondingly larger volume, and that its weight was diminished by that of the air which it displaced, which was exactly equal to the air which it contained. Aristotle's experiment was regarded as decisive for many centuries ; and it was not till the time of Galileo¹ that the air was known to be a heavy fluid. It was reserved, however, for later philosophers to see in this fact the true cause of a variety of phenomena which were previously unexplained.



le¹ that the air was known to be a heavy fluid. It was reserved, however, for later philosophers to see in this fact the true cause of a variety of phenomena which were previously unexplained.

Otto Guericke² is said to have devised the following experiment for showing that the air has weight :—

By means of the air-pump, of which he was the inventor, he exhausted a glass globe, fitted with a stopcock, of its contained air. He then very carefully weighed the globe, and as soon as the scalebeam was perfectly horizontal he opened the stopcock. The air immediately rushed in and the globe descended. From this he very justly inferred that the additional weight which had to be placed in the other scalepan to restore equilibrium was equal to the weight of the quantity of air admitted into the globe.

§ 91. **Measure of Atmospheric Pressure. Torricelli's Experiment.**—To Torricelli is due a most important experiment, which not only shows us that the air has weight, but also gives us a measure of its pressure-intensity at any point.

¹ Born at Pisa 1564 ; died 1642. ² Of Magdeburg ; 1602–1686.

Take a glass tube about 34 inches in length, open at one end and closed at the other. Fill it carefully with mercury, and, placing the thumb over the open end, invert the tube with this end under the surface of some mercury contained in a cup. On removing the thumb the mercury will be found to sink somewhat, and after a time will remain stationary in the tube, with its surface-level about 30 inches above the surface-level of the mercury in the cup.

FIG. 47.



Now, since the pressure-intensity is the same at all points in the same horizontal plane of a liquid in equilibrium, the pressure-intensity at any point of the external surface of the mercury is equal to that along $c d$. But the pressure-intensity on $c d$ is equal to $h s$, where h is the difference of level of the mercury inside and outside the tube, and s is the weight of a unit volume of mercury. Hence the intensity of the atmospheric pressure on the external surface of the mercury is $h s$; and the pressure on any area of the same size as the section of the tube is equal to the weight of a column of mercury having that sectional area for base and the difference of level of the surface-level of the liquid inside and outside the tube for height.

The space above the surface of the mercury in the tube is called the Torricellian vacuum. It is really occupied by mercury vapour, the effect of which on the height of the column may generally be disregarded.

§ 92. **Effect of Atmospheric Pressure.**—The downward pressure exerted by the atmosphere is the

real cause of a number of phenomena which, at one time, were explained on the supposition that 'Nature abhors a vacuum.'

It was observed that whenever a fluid was forcibly removed from a vessel, air or some other fluid took its place; and this tendency to occupy a vacant space was found to be sufficiently strong to counteract gravity and

FIG. 48.



other forces. Thus if the bulb of a glass vessel, the stem of which is under water, be slowly heated, bubbles of air will rise through the water, owing to the expansion of the air in the bulb; and if the source

of heat be afterwards removed, the volume of the air will gradually diminish, and the water will rise in the stem. The same effect is produced in a tube open at both ends by ordinary suction.

From observations such as these it was inferred that Nature abhors a vacuum; and this general proposition, which expresses, though somewhat vaguely, the results of experiments made, within certain limits and under particular conditions, was supposed for many centuries to constitute an undeniable law of Nature. Galileo was the first to show that this so-called law was not universally true. He found that water would not rise in an empty tube to a greater height than about 33 feet, and he very rightly concluded that the force supporting this column of water was the pressure of the atmosphere. Nature's abhorrence of a vacuum was thus proved to have a practical limit. Galileo does not, however, appear to have recognised all the consequences of his own

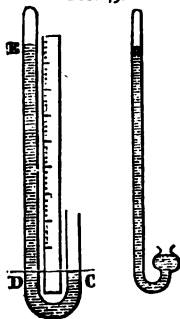
discovery. It was left to Pascal, by an extension of Torricelli's experiment, to give to this important principle its full development. By performing Torricelli's experiment at different elevations above the sea-level, Pascal successfully disproved the old theory, which he showed was the result of insufficient and circumscribed observations ; and by clearly establishing the fact that the column of liquid supported varied with the height of the atmosphere above it, he proved conclusively that the atmospheric pressure was the true cause of all the phenomena in question.

§ 93. **Barometers.**—A barometer in its simplest form consists of a tube such as that used in Torricelli's experiment, and containing a column of liquid supported by the atmospheric pressure which it serves to measure. The height of the column is the difference in level between the surface of the liquid in the tube and in the cup. This height varies with the liquid employed and the conditions of the atmosphere. The great specific gravity of mercury renders that liquid best adapted for the construction of a barometer, as the height of the column is correspondingly small. With a mercury barometer the average height is 76 cm.

A form of instrument very commonly in use is that known as the *siphon barometer*.

It consists of a bent tube open at one end and closed at the other. Mercury having been introduced into the tube in sufficient quantity to

FIG. 49.



fill the longer leg, the tube is placed in a vertical position when the mercury assumes a position of equilibrium, as shown in fig. 49.

Its principle is the same as that of the instrument already described. The pressure-intensity at D, due to the column of mercury ED, is equal to that at C, due to the column of air above it.

In this form of instrument the cup containing mercury is not required; the difference of level of the surfaces of the mercury in the two branches of the tube measures the pressure due to the atmosphere.

§ 94. **Barometric Corrections.—Absolute pressure per unit area.**—When the barometer is used for determining differences of atmospheric pressure at different places, or at the same place at different times, certain corrections must be made in the observed height of the column, of which the following are the principal:—

1. *Correction for Temperature.* — A column of mercury that measures 76 cm. at 0° C. is found to have increased in length when measured at some higher temperature, in consequence of the expansion of the liquid under the influence of heat. It is necessary, therefore, in comparing barometric heights at different places and at different temperatures to calculate what the height of the column at each place would have been if the temperature had been the same; and it is usual to reduce the observed height of the column in each case to the height of a column that would produce the same pressure at 0° C. In order to obtain accurate results, a further correction must be made for the expansion of the scale on which the measurements are marked.

2. *Correction for Capillarity.*—When the mercury is contained in a narrow tube, the internal diameter of which is less than three-quarters of an inch, the column is, as we have seen, sensibly depressed in consequence of the tension of the convex surface of the mercury. In this case the barometric reading, as reckoned from the top of the convex meniscus to the surface-level of the mercury in contact with the air, indicates a pressure a little less than that of the atmosphere, and the necessary correction must be added. The error due to capillarity is only observable in narrow tubes, and is slightly different when the mercury is rising from what it is when the mercury is falling in the tube.

3. *Correction for Difference of Sea-level.*—Owing to the compressibility of air its density at places near the sea-level is greater than at places higher up. The exact law according to which the density of the air decreases as we ascend will be considered later on ; but even if the density of the atmosphere were uniform, the pressure-intensity would be greater at places of low than of high elevation. In fact, if a barometer is carried up a mountain it indicates a continuously decreasing atmospheric pressure ; and, from the difference in height of the barometric column at two places, the difference of the elevation of the two stations above the sea-level can be determined.

But the height of the barometric column is liable to frequent fluctuations, owing to occasional and accidental causes, the effect of which it is often required to ascertain. If, with this object, observations of the barometer are made at two stations at different heights, an addition must be made to the observed reading at

each place, so as to reduce it to what it would be at the sea-level. Thus, if the barometric column falls, on the average, 60 mm. in being carried from the sea-level to station A, and 85 mm. to station B, 60 mm. and 85 mm. must be added to the observed reading at each of those stations.

4. *Correction for the unequal value of g.*—In estimating the pressure of the air on a given area, by the weight of a column of mercury, we obtain a result which varies with the value of g , and affords, therefore, no uniform standard for the comparison of the atmospheric pressure in places situated in different latitudes. At Paris, where $g = 980.94$ cm., the pressure represented by a mercury column of the same height is less than at Greenwich, where $g = 981.17$ cm. If h equals the height of the barometric column, and d the density of the liquid employed, the measure of the pressure of the air per unit-area is $g h d$, absolute units of force. If we put $h = 76$ cm., and $d = 13.596$, the density of mercury, then the absolute pressure at Greenwich per square centimetre is

$$981.17 \times 76 \times 13.596 = 1.0138 \times 10^6 \text{ CGS units of force.}$$

It would be practically convenient to take the round number 10^6 units of force per square centimetre as the standard of atmospheric pressure. This pressure would be represented at Greenwich by 74.964 cm., or 29.514 inches of mercury.

The standard atmospheric pressure is commonly called 'an atmosphere'; and a pressure twice as great is known as 'two atmospheres,' and so on. Thus, a pressure of 10^7 units of force would be a pressure of ten atmospheres.

§ 95. **Examples.**—(1.) Required the height of a water barometer when the mercury barometer stands at 30 inches, the density of mercury being 13·596.

Let h be height of water barometer,
 h' ,, ,, mercury barometer,

and let d and d' be densities of water and mercury respectively. Then, atmospheric pressure per unit area = $g h d = g h' d'$ and

$$\therefore h = 30 \times 13 \cdot 596 = 34 \text{ ft. nearly,}$$

since $d = 1$ and g is constant.

(2.) If a mercury barometer standing at 76 cm. be immersed 4 metres below the surface of a lake, find the height of the column.

A pressure of 4 metres of water is equivalent to a pressure of $4 \div 13 \cdot 596$ metres of mercury, *i.e.*, 29·4 cm. nearly. Hence the increase of pressure will be denoted by a rise of 29·4 cm., or the height of barometric column will be 105·4 cm.

(3.) What is the absolute pressure, in units of force, due to a height of 100 metres of sea-water of density 1·027, g being 981?

The pressure equals $981 \times 1 \cdot 027 \times 100 \times 100 = 1 \cdot 0075 \times 10^7$ units of force per sq. cm., and is equal to about 10 atmospheres.

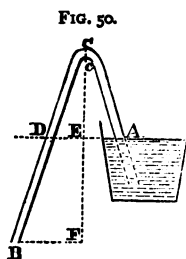
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§ 96. **The Siphon.**—This is an instrument the action of which depends on the atmospheric pressure. It is used for drawing off liquid from one vessel to another, and consists of a bent tube with arms of unequal length.

The siphon must be first filled with the liquid to be drawn off, and the shorter arm being temporarily closed, and then plunged beneath the liquid, a continuous flow will take place.

The action is thus explained :—

Consider cc a vertical element of fluid occupying the section of the tube at its highest part.



Then, if AD be a horizontal plane continuous with the surface of the liquid, the pressure on the side AC of the element cc is

$$H - CE,$$

where H is the height of a column of the same liquid, corresponding to the pressure-intensity of the atmosphere; and the pressure on the other side is $H - CF$.

Hence, as CF is greater than CE , there is a resultant force acting from right to left, and causing the liquid to flow along CD . In this way a continuous flow is maintained. It is of course understood that CE is not greater than H ; for if CE were greater than H , the liquid would commence to flow back along the limb CA . Whatever cause may diminish H diminishes the maximum height over which the liquid can be carried. Should the surface-level of the liquid in the vessel fall below B the direction of the resultant force would be changed, and the flow would consequently cease.

The resultant force acting on element cc is the pressure represented by the column of liquid

$$H - CE - (H - CF) = CF - CE = EF;$$

and, hence, the velocity of efflux is $\sqrt{2g EF}$, neglecting friction, and supposing the pressure at B to equal the pressure at A .

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EXERCISES. IX.

1. If in ascending a mountain the barometer falls from 76 cm. to 51 cm. find the decrease in pressure on an area of one square metre.

2. A mercury barometer stands at 29.5 in., and the specific gravity of mercury is 13.6 : find the specific gravity of oil, if a column 36 ft. 6 in. in height can be supported by the atmospheric pressure alone.

3. When the ordinary mercury barometer stands at 30 in. find the whole atmospheric pressure on a surface the area of which is 10 square feet.

4. At what depth below the surface of a lake will the barometer indicate a pressure of 50 in., when the pressure of the atmosphere is 30 in. ?

5. In a siphon barometer of uniform bore the level of the mercury in the open end falls through 4 mm.: what change of pressure does this indicate ?

6. If the sectional areas of the open and closed branches of a siphon barometer are as 4 to 1, through what distance will the mercury move in the closed branch, if the mercury in the ordinary barometer rises one inch ?

7. If the specific gravity of air is 0.0013 when the barometer stands at 76 cm., find its sp. gr. when the barometer stands at 58 cm.

8. Supposing the average barometric height to be 30 in., the sp. gr. of mercury 13.6, and of air 0.0013, find the height of the atmosphere, supposing the density to be uniform throughout.

9. A barometer is observed to fall $\frac{1}{10}$ of an inch when carried up 88 feet of vertical height: how much would it fall if taken 110 yards up a hill rising 1 in 3 ?

10. If the specific gravity of mercury is 13.6, what ought to be the length of a water barometer inclined to the horizon at an angle of 60° , the mercury barometer standing at 30 ins. ?

11. If the height of the column in an ordinary barometer is h , and the tube is inclined through an angle of 30° from the vertical, what will be the barometric reading ?

12. A small bead of glass floats in the mercury of an ordinary barometer : does it affect the barometric reading ?

13. Find the greatest height over which water can be carried by a siphon at the top of a mountain where the mercury barometer stands at 50 cm.

14. Over what height can a liquid whose sp. gr. is s be carried by a siphon, when the height of the mercurial barometer is h , and sp. gr. of mercury is s' ?

15. A cylindrical body 40 inches high floats in water at the ordinary atmospheric pressure with 10 inches of its height immersed, the sp. gr. of air being .0013. The body, with the vessel in which it floats, is then placed under a bell in which the atmospheric pressure is ten times as great : what part of the body will now be immersed ?

16. If the atmosphere press with 15 lbs. on every square inch, and the weight of a cubic foot of water be $62\frac{1}{2}$ lbs., find the total pressure on a rectangular plane surface, placed vertically with its upper edge a foot deep in water, and its lower edge 3 ft. deep, the rectangular surface being 6 ft. long and 2 ft. wide.

17. Find an equation for determining the internal radius of a globe of thickness t and specific gravity c , which will just float in air of specific gravity s , when filled with gas the density of which as compared with air is d .

18. Taking the pressure of the atmosphere at 15 lbs. per square inch, the height of the salt-water barometer at 30 feet, calculate the pressure-intensity at 50 fathoms depth in tons per square foot. If this pressure acted for a second on a square yard of the stern of a vessel weighing 300 tons, what velocity would it communicate, there being no resistance to the motion ?

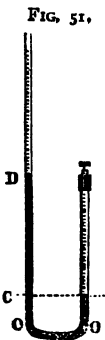
XV. Relation between the pressure and volume of a gas, the temperature remaining constant. — Boyle's Law.

We have seen that the characteristic quality of a gas is its expansibility and compressibility. We come

now to consider the quantitative relation that exists between change of volume and change of pressure, the temperature remaining constant. This relation can be inferred from a series of experiments, of which the following are instances :—

§ 97. **Experiments.**—*For pressures greater than the atmospheric pressure.*—1. A simple form of apparatus for these experiments consists of a uniform bent tube, having one branch open at the top and considerably longer than the other, which is closed. The shorter branch is sometimes fitted with a screw-cap so that the pressure of the enclosed gas may be more easily regulated. The tube itself is graduated, or a scale is fixed to each branch.

If the cap, which should fit air-tight, be first unscrewed, and some mercury poured into the tube, the mercury will rise to the same level in both branches. If now the cap be screwed on again some air will be enclosed under a pressure equal to that of the external atmosphere. Call this pressure 76 cm., and suppose the mercury to stand at 0 in each branch. Pour more mercury into the tube, and the level of the mercury will be lower in the shorter than in the longer branch. The volume of the air confined in the shorter branch is now diminished, and its pressure is increased by that of the column *c d*.



Suppose the air in the tube to have originally occupied 20 divisions of the tube, or 20 cm., and that mercury has been poured into the tube to a height of

23 cm. above its former level in the longer branch, then the mercury in the shorter branch will stand at 4 cm. above 0, and, consequently, the enclosed air will occupy only 16 cm. The increase of pressure, due to the difference of level, is therefore $23 - 4 = 19$ cm.

Now, if we examine these numbers we shall find

$$20 : 16 :: 19 + 76 : 76$$

$$\text{i.e.} \quad \quad \quad :: 95 : 76,$$

$$\text{or, } 20 \times 76 = 16 \times 95,$$

which shows that the variation in the volume is inversely as the variation in the pressure ; or, what is the same thing, that the product of the volume into the pressure is constant.

V	H	h	$H-h$	$H-h+75.9=P$	$P V$
15.0	34.1	7.3	26.8	102.7	1540
12.0	62.4	10.0	52.4	128.3	1539
10.0	90.1	11.8	78.3	154.2	1542
9.1	106.0	12.7	93.3	169.2	1539
8.3	121.3	13.3	108.0	183.9	1526
7.4	147.0	14.1	132.9	208.8	1545
6.7	167.5	14.8	152.7	228.6	1547
6.4	180.2	15.0	165.2	241.1	1537
Mean value . . . 1539.3					
V =volume of air in cub. cents. ; H =height in cents. of mercury in open limb ; h =height in closed limb ; 75.9 = height of barometer.					

On comparing the results of a number of actual experiments, the value of the product of the volume into the pressure will be found to vary slightly, the extent of the variation depending on the degree of

accuracy with which the experiments are performed. In the above table are found the results of a few experiments in which the measurements were roughly made, the fractional parts of a centimetre being estimated by eye. They serve, however, to show how nearly constant is the value of $P V$.

2. *For pressures less than the atmospheric pressure.* To determine the relation between the change in the volume and pressure of a gas, when the gas expands, we take a barometer tube fitted with a screw-cap, and having opened the tube, partly immerse it in a vessel containing mercury. We then close the tube, by means of the screw, and thereby enclose a certain quantity of air at the ordinary atmospheric pressure. The tube is now raised vertically upwards, and the enclosed air expands, its pressure being diminished by that due to the difference of surface-level of the mercury in the tube and in the vessel. Thus, if $ab = V$ be the volume occupied by the air when the mercury stands at the same level in the tube and in the vessel—that is, under the ordinary atmospheric pressure H , and if $ad = V'$ be the volume occupied at the pressure $H - cd = P'$, then, on reference to the scale, it will be found that

$$ab : ad :: H - cd : H,$$

or, $VH = V'P'$.

FIG. 52.



§ 98. **Statement of Boyle's¹ Law.**—From experi-

¹ Robert Boyle born at Lismore in Ireland, 1626 ; died 1691.

ments such as these performed with different gases, and at different temperatures, a law has been established, which we shall see is not accurately true for any gas, but approximates very nearly to the truth for those gases which are not easily reducible to the liquid state. The law is known as Boyle's law or Mariotte's law, as these two philosophers are said to have arrived independently and by similar experiments at the same result. It may be enunciated thus :—

The volume of a gas varies inversely with its pressure, when the temperature remains constant.

Thus, if the volume V change to V' , while the pressure changes from p to p' , we have :—

$$V : V' :: p' : p ; \text{ or } Vp = V'p'.$$

As the mass of the gas remains the same in the experiments already described, it follows that the density of the gas must increase as the volume diminishes, and *vice versâ*. Hence Boyle's law may be stated thus :—

The pressure of a gas is proportional to its density, the temperature remaining constant.

For a perfect gas, therefore, we have the following relations :—

$$\frac{V}{V'} = \frac{p'}{p} = \frac{d'}{d}.$$

Another statement of this law, due to Professor Rankine, places the law in a very clear light :—

‘If we take a closed and exhausted vessel and introduce into it one grain of air, this air will, as we know, exert a certain pressure on every square inch of the surface of the vessel. If we now introduce a

second grain of air then this second grain will exert exactly the same pressure on the sides of the vessel that it would have exerted if the first grain had not been there before it. Hence we may state, as the property of a perfect gas, that any portion of it exerts the same pressure against the sides of a vessel as if the other portions had not been there.¹

§ 99. **Dalton's Law.**²—This is an extension of Boyle's law for a mixture of different gases. If several different gases, which do not act chemically on one another, are placed in a vessel, the pressure on the sides of the vessel is the sum of the pressures due to the different gases. Thus, suppose each of the gases, if in the vessel by itself, to exert pressures the intensities of which are $p_1, p_2, p_3, \&c.$, respectively, the intensity of the whole pressure exerted by the mixture is $p_1 + p_2 + p_3 + \&c.$ Rankine's statement of Boyle's law shows this to be the case for different parts of the same gas; hence Boyle's law may be considered as a particular case of Dalton's,³ which may be thus stated:—*When a mixture of several gases, at the same temperature, is contained in a vessel, each produces the same pressure as if the others were not present.*

§ 100. **Examples.**—(1.) In a bent tube, open at one end and closed at the other, the mercury stands at the same level in both branches, and the contained air occupies 30 cm. at the normal atmospheric pressure—viz. 76 cm. If the section of the tube is 10 sq. cm., what volume of mercury must be poured into the tube to compress the air to 20 cm. ?

If p be the pressure of the air when occupying 20 cm., we

¹ C. Maxwell, *Theory of Heat*, pp. 27–8. ² 1801.

³ Born in Cumberland 1766; died 1844.

have by Boyle's law $20 \times p = 30 \times 76$, $\therefore p = 114$, of which 76 is due to atmospheric pressure. Hence the difference of level in the two branches equals 38 cm.; and as the mercury has risen 10 cm. in the closed branch the quantity of mercury introduced equals $(20 + 38) 10 = 580$ cubic cm.

(2.) The mercury in an ordinary barometer stands at 30 in., and the sectional area of the tube is one square inch. A cubic inch of air is admitted through the mercury into the vacuum above, and depresses the column through 4 inches: find the size of the vacuum.

When the cubic inch of air is admitted into the vacuum it first fills the vacuum, and then with its diminished elasticity, consequent on its expansion, it depresses the mercury until the pressure of the column of mercury, together with the elasticity of the air, equals the atmospheric pressure.

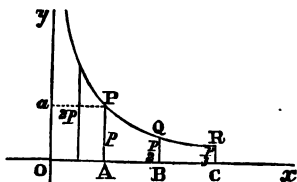
Suppose the vacuum to measure x inches; then the air which under a pressure of 30 inches occupied 1 inch is found to occupy $x + 4$ inches under a pressure of $30 - (30 - 4) = 4$ in.

Hence, by Boyle's law, $4 \times (x + 4) = 30$, or $x = 3.5$ in.

§ 101. Graphic representation of Boyle's Law.—

Take ox , oy , two straight lines at right angles to each

FIG. 53.



other, and along ox mark off OA , to represent the number of units (V) of volume which a certain quantity of gas occupies at a pressure p . Then, if we mark off OB , OC . . . to

represent $2V$, $3V$. . . we know by Boyle's law that the pressures corresponding to these volumes will be

$\frac{p}{2}$, $\frac{p}{3}$ Hence, if AP be drawn vertically from

A to represent p , then the lines $BQ = \frac{AP}{2}$, $CR =$

$\frac{AP}{3}$, &c. will represent respectively $\frac{p}{2}$, $\frac{p}{3}$, &c., and the curve drawn through these points will serve as a graphic representation of Boyle's law.

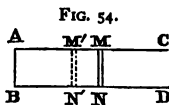
Now, it is evident that no amount of compression can reduce the volume of a gas to zero, and no amount of expansion can wholly destroy the pressure which a perfect gas exerts; it follows, therefore, that the curve will approach on either side nearer and nearer to the lines ox and oy , but will never meet them.

Since, also, $Vp = V'p' = V''p''$, we see that the product of the volume and the pressure remains constant, whilst the separate factors vary, and therefore the curve PQR is such that if P be any point in it, the area of the rectangle $OAPa$ is constant for all positions of P .

Now, if x represent any volume measured along ox , and y the corresponding pressure measured vertically upwards, we have $xy = a$ constant, and the curve corresponding to this equation is known as the rectangular hyperbola.

§ 102. Graphic representation of the work done in changing the volume of a gas from V to V' , the temperature remaining constant.

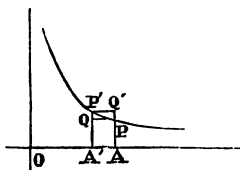
Let a volume V of gas be enclosed in the cylindrical vessel $DBAC$ by the piston MN , and let the piston be moved through MM' so that the volume becomes V' . Then if p be the pressure of the gas, when the volume is V , the work done in changing the volume to V' would be $p \times MM'$,



supposing p to remain the same throughout. But this is not the case, for the pressure increases as the volume diminishes. If, therefore, p' is the pressure of the gas when its volume becomes V' , the work done is equal to some quantity the value of which lies between $p \times MM'$ and $p' \times MM'$.

Now, suppose OA (fig. 55) represent the volume V , and OA' the volume V' , AP , $A'P'$ the correspond-

FIG. 55.



ing pressures; then, if we draw the lines PQ , $P'Q'$, parallel to OA , the work done is represented by the area of a figure which is greater than the rectangle $PQA'A$, and less than the rectangle $P'A'AQ'$. Now,

by reasoning similar to that employed for finding the space described when a body moves with an increasing velocity, it can be shown that the area of the figure $P'A'A$ represents the work done.

§ 103. **Limits of Boyle's Law.**—By experiments conducted by Regnault¹ and by Despretz² it has been observed that Boyle's law is not perfectly true for any actual gas. For air and all gases that do not readily liquefy under pressure the law is found to be a very close approximation to the results of experiments; but for easily liquefiable gases, such as carbonic dioxide and ammonia, the volume is found to decrease more rapidly than the pressure increases; and the divergence of the law from the true results is greater as the gases approach their point of liquefaction.

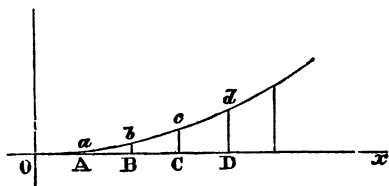
¹ 1827, 1847.² 1827.

Thus carbonic-dioxide under a pressure of twenty atmospheres is found to occupy only four-fifths of the volume given by the law.

Regnault devised very careful experiments for showing the exact relation between the pressure and volume of different gases and the extent of the divergence of this relation from Boyle's law. He arrived at the following result: that whereas, according to Boyle's law, $Vp = V'p'$, or $\frac{Vp}{V'p'} - 1 = 0$, the expression $\frac{Vp}{V'p'} - 1$ is a quantity having a small positive value for all gases except hydrogen, and increases gradually with the pressure.

If the distances OA , OB , OC , measured along the horizontal line ox represent pressures of one, two,

FIG. 56.



and three atmospheres, and if the vertical lines Aa , Bb , Cc represent the values of $\frac{Vp}{V'p'} - 1$ for these pressures, the curve formed by joining $oabc\dots$ is a graphic representation of the divergence of the results of direct experiment from Boyle's law.

Seeing that $\frac{Vp}{V'p'} - 1$ is a positive quantity, Vp

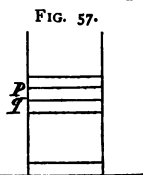
must be greater than $V' p'$, and therefore the value of V' must be less than it would be if $V p = V' p'$, in accordance with Boyle's law. Hence it appears that gases are more compressible than is in accordance with Boyle's law.

In this respect, however, hydrogen differs from other gases, and this exception shows that the law is more complicated than it would seem to be even with the extension above given. It has, however, been proved that the true relation of the volume of a gas to its pressure depends to some extent on the temperature. The curve exhibited in fig. 56 is not the same for all temperatures. For nearly all gases the value of the expression $\frac{V p}{V' p'} - 1$ continually *decreases* as the temperature rises, and we are 'thus led to expect that if the temperature were sufficiently high this quantity would pass through zero and become negative; or the curve, after coinciding with the line ox , would reappear on the other side. It should seem, therefore, that at a certain temperature varying with each gas Boyle's law is strictly accurate, and that for higher temperatures the law of the divergence is changed, so that the density increases less rapidly than the pressure. Now, as hydrogen which has already been reduced to the solid state is commonly supposed to be the vapour of a metal, it is relatively at a very high degree of rarefaction, and this fact may be the reason why $\frac{V p}{V' p'} - 1$ has a negative value for this gas.

§ 104. Relative Densities of the Air at different

Heights.—We are now in a position to determine the law according to which the density of the air changes as we ascend from the level of the sea. The decrease in the density of air is owing to its compressibility; but even if the air were as incompressible as water, and the atmosphere were homogeneous throughout, the barometric column would be found to fall, in rising from places of lower to places of higher elevation, in consequence of the diminution in the height of the column of air. In order to obtain an approximate relation between the densities of the air at two different heights we must neglect the accidental differences of pressure caused by differences of temperature and moisture, and by the altered value of the force of gravitation.

Take a vertical column of the atmosphere and suppose it divided by horizontal planes into a number of strata so thin that the density for each layer may be considered uniform and equal to that at its lower surface. Let the height of the column be z , and let n be the number of



strata, so that the thickness of each layer is $\frac{z}{n}$

let the section of the column be the unit of area.

Let d and p be the density and pressure of the atmosphere at the surface of the earth, and let $d_1, d_2, d_3 \dots, p_1, p_2, p_3 \dots$ be the densities and pressures at the successive levels.

Then, since the difference between the pressures at the upper and lower surface of a layer must be equal to the weight of that layer of air,

$$p - p_1 = \text{weight of lowest layer} = dg \frac{z}{n} \quad (\S 17):$$

And since by Boyle's law the pressure is proportional to the density, we have $\frac{p}{d} = \frac{p_1}{d_1} = \frac{p_2}{d_2} =$

$$k (d - d_1) = a$$

whence
$$d_1 = \left(1 - \frac{g}{k} \right)$$

similarly
$$d_2 = \left(1 - \frac{g}{k} \frac{z}{n} \right)$$

$$d_3 = \left(1 - \frac{g}{k} \frac{2z}{n} \right)$$

and
$$d_n = \left(1 - \frac{g}{k} \frac{z}{n} \right)$$

i.e., the ratio of the densities for any two stations is constant, since g , k , z and n are constant. Therefore, the quantities d , d_1 , d_2 , d_3 , ... form an arithmetical progression, the heights of the stations forming an arithmetical progression.

If, therefore, d_n be the density

$$\frac{d_n}{d} = \left(1 - \frac{g}{k} \frac{z}{n} \right)$$

§ 105. To find the difference in heights of two stations by means of a barometer, the temperature and force of gravity being constant.

Let H and h represent the heights of the two stations, the vertical distance being z ,

Then $H : h :: d : d_n$

$$\frac{h}{H} = \left(1 - \frac{g}{k} \frac{z}{n} \right)$$

where d is the density of the air at the lower, and d_n the density at the upper station. Now, if n increase without limit, it is proved in works on Algebra, that the value

of the right-hand side of this equation becomes $e^{-\frac{g}{k} \cdot z}$

$$\text{Hence } \frac{H}{h} = e^{\frac{g}{k} \cdot z}, \text{ and } \log_e \left(\frac{H}{h} \right) = \frac{g}{k} \cdot z$$

$$\text{Or, } z = \frac{k}{g} (\log_e H - \log_e h)$$

EXERCISES. X.

1. A gas occupies 100 litres when the barometer stands at 76 cm.: find the increase in the volume of the gas, if the pressure becomes 73 cm.

2. A tube 2 feet long is filled with water and inverted in a vessel of water, with its open end below the surface. Air at a pressure of 30 ins. is then admitted into the tube till the level of water in the tube is the same as that outside, and the air occupies 12 inches. The tube is now raised till the air occupies 15 inches: find the pressure of the contained air.

3. Into the vacuum above a common barometer, which stands at 30 inches, 2 cubic inches of air are admitted, which depresses the mercury 6 inches: if the section of the tube is one square inch, find the size of the vacuum.

4. A vessel of 3 cubic feet capacity containing air at two atmospheres' pressure is put into communication with a vessel of 18 cubic feet capacity containing air at $\frac{1}{4}$ of the atmospheric pressure: what is now the pressure of the air in the two vessels?

5. A horizontal cylinder containing air is fitted with a piston which is 10 inches from the closed end when the pressure is 15 lbs. on the square inch. If the area of the piston is 8 square

inches, find the force that must be exerted to hold the piston at a distance of 12 inches from the closed end.

6. If the water barometer stand at 33 feet, to what depth must a small cylindrical vessel be sunk to reduce the volume of the contained air to one-third of its original volume, the height of the vessel itself being neglected?

7. A cylinder contains air at the ordinary atmospheric pressure, and is fitted with a piston (area 10 square inches), which is 1 ft. from the closed end. If the cylinder be set vertically with its open end upwards, how far will the piston descend, the atmospheric pressure being 15 lbs. on the square inch, and the piston weighing 10 lbs.?

8. Suppose the cylinder is held with the open end downwards, how far will the piston fall?

9. Find what weight must be hung to the piston in the last question to draw it down 2 inches from its original position.

10. The air contained in a cubical vessel the edge of which is one foot, is compressed into a cubical vessel, the edge of which is one inch: compare the pressures on the side of each vessel.

11. Forty c.c. of air are enclosed in a tube over mercury, the height of the mercury in the tube above the level in the vessel outside being 50 cm. ($cd = 50$, fig. 52). The tube is depressed until $cd = 30$ cm. What is now the volume of the air, the height of the barometer being 76 cm.?

12. A bent tube (fig. 51) has a uniform section of 1 square inch and is graduated in inches; 6 cubic inches of air are enclosed in the shorter branch, when the mercury is at the same level in both branches. What volume of mercury must be poured into the longer branch in order to compress the air into 2 inches? The barometer stands at 30 ins.

13. The air enclosed in the shorter branch of a similar bent tube occupies 11.3 c.c., and the difference of level in the two branches is 60.2 cm. If the barometer stands at 75.9 cm., find what volume the air would occupy under the atmospheric pressure only.

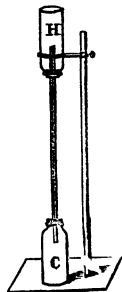
XVI. *Diffusion of Gases.*

§ 106. **Diffusion of Gases. Experiments.**—We have seen that if two layers of different liquids are in contact with each other, or are separated by a porous diaphragm, a mixture of the liquids takes place, the one diffusing into the other. Now, the same phenomenon is observable with gases; but the laws of gaseous diffusion are less complex than those of liquid diffusion, in consequence, probably, of the greater structural simplicity of gaseous bodies.

Fill two jars with two different gases—for example, with chlorine and hydrogen—and let the jars be connected by a long tube, that containing the hydrogen or lighter gas being placed uppermost. In a few hours the chlorine will find its way into the upper jar, as may be seen by its green colour; and the hydrogen will take its place. Each of the jars will be found to contain the same proportion of the two gases, and the gases will remain permanently mixed. This intermixture takes place between any gases or vapours which do not act chemically on one another.

If a vessel containing nitrogen be covered with a porous diaphragm of some colloid substance, and be placed under a bell-jar containing hydrogen, diffusion will take place, and after a time the membrane will have become convex, showing that the hydrogen has been passing inwards more

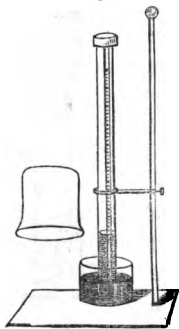
FIG. 58.



rapidly than the nitrogen has passed outwards. If the position of the gases be reversed the contrary result will be found. If the diaphragm be moist, and one of the gases is soluble, its rate of diffusion is very much increased. Thus, if a moist thin bladder be distended with air and placed in a bell-jar containing carbonic dioxide, this gas, owing to its solubility, passes much more rapidly into the bladder than the air escapes from it, and very frequently breaks it, though the rate of diffusion of carbonic dioxide into air is really less than that of air into the gas.

§ 107. **Rate of Diffusion.**—If we take a long graduated tube, open at both ends, and close one end by a plug of porous clay, and immerse the open end in a vessel containing mercury, or water coloured, for the sake of greater distinctness, the level of the liquid in the tube and in the vessel will be the same, showing that the gas must be entering the tube through

FIG. 59.



the porous plug at the same rate as it escapes into the outer atmosphere. If now we bring an inverted beaker filled with coal-gas over the closed end of the tube, we find that there is a bubbling of gas through the liquid, clearly showing that the coal-gas is entering the tube more rapidly than the air is escaping from it. As soon as we withdraw the beaker the liquid rises in the tube, the pressure of the mixed gas in the tube being less than that of the atmosphere outside ; and if we replace the beaker

the bubbling of the gas through the liquid recommences. This experiment enables us to observe the difference only between the amount of coal-gas that goes in and of air that escapes from the tube. We can vary this experiment by filling the tube with other gases than air. Suppose the tube to be first filled with hydrogen and then inverted in the liquid, and held in such a position that the level of the liquid within and outside the tube is the same, the beaker being altogether removed. After a time the liquid will be found to rise in the tube, showing that the pressure within the tube has become diminished, and that the hydrogen must be passing from the tube into the external air more rapidly than the air is entering the tube.

It is easy to see that, if the tube be graduated, careful experiments will show the volumes of different gases that diffuse into the air in the same time. If the beaker be filled with a different gas, and held over the tube, the rates of diffusion of different gases into each other can be ascertained.

By experiments such as these, Graham established the law that *the rates of diffusion of two gases into each other are in the inverse ratio of the square-roots of their densities*. For instance, taking the density of air as *unity*, that of hydrogen is 0.0692; and the square-roots of these numbers being 1 and 0.2632 respectively, the law tells us that the rate of diffusion of hydrogen is to the rate of diffusion of air

$$\text{as } 1 : 1 \div 0.2632, \text{ i.e. } :: 1 : 3.7994;$$

and actual experiment shows that whilst one measure

of air passes into the tube containing hydrogen, 3·83 measures of hydrogen escape into the air.

This important law can be further verified by taking a vessel consisting of two large receivers filled with different gases, and connected by a tube with a stop-cock. If now the gases be allowed to diffuse into each other for a certain period of time, the contents of the receivers can be analysed, and the proportion of the two gases in each can be quantitatively determined. By varying the time, and tabulating the results, the accuracy of the law may be verified for any two gases.

The following table shows the results of some of Graham's experiments :—

Gas	Density	Square root of density	$\frac{1}{\sqrt{\text{Density}}}$	Rate of diffusion
			$\frac{1}{\sqrt{\text{Density}}}$	
Hydrogen	0·06926	0·2632	3·7994	3·83
Marsh-gas	0·559	0·7476	1·3375	1·34
Carbonic Oxide . .	0·9678	0·9837	1·0165	1·0149
Nitrogen	0·9713	0·9856	1·0147	1·0143
Oxygen	1·1056	1·0515	0·9510	0·9487
Nitrous oxide . . .	1·527	1·0914	0·8092	0·82

If d and d' be the densities of two gases, and D and D' the volume of each which diffuses into the other in the same time, then, according to the law,

$$D : D' :: \frac{1}{\sqrt{d}} : \frac{1}{\sqrt{d'}}$$

$$\text{Or } D^2 d = D'^2 d'.$$

§ 108. **Kinetic Theory of Gases.**—In order to explain the diffusion of gases we must suppose that the

particles of a gas are independent of one another, that they are constantly moving in all directions, and with a very great velocity. These particles during their motion frequently impinge on one another, and the direction of their motion is consequently changed. When they come into contact with the sides of the vessel containing the gas their momentum is resisted ; and it is to this shower of particles moving with a considerable velocity that the pressure of a gas is supposed to be due. The notion that the particles of a gas are in rapid motion, and that it is by their impact that gases press on one another, is a very old one. It was pointed out, not long after Newton's time, by Daniel Bernouilli¹ ; Lesage and Prevost of Geneva made several applications of the theory, and it was afterwards revived in this country by Herepath. In 1848 Dr. Joule showed how the pressure of gases might be explained by the impact of their molecules, and he calculated the exact relation that exists between the observed pressure of a gas and the velocity of its particles. It is, however, to Professors Clausius and Clerk-Maxwell that this theory of the molecular structure of gases owes its chief development.

By a method similar to that indicated in Wormal's 'Thermodynamics,' § 71, Joule showed that if a vessel contain hydrogen at the ordinary pressure and at 0° C., the velocity of the particles must be about 6,055 feet per second. Now, although the velocity of these particles is so considerable, the number of particles occupying a given volume, say a cubic inch, is so enor-

¹ Born at Groningen 1700 ; died 1782.

mous that the particles move through a very small space, and are unable to travel from side to side of the vessel containing them, without encountering a series of successive impacts with other particles. It follows from this, that if the average velocity of the particles remains the same, the pressure exerted at any point of the vessel containing the gas depends on the number of particles that impinge, in a given time, on the element of area containing that point. But this, of course, depends on the number of particles contained in the vessel, *i.e.*, on the density of the gas; for if the volume of the gas be doubled, the average velocity of the particles remaining the same, the number of particles traversing a given area will be halved. Now, the density of a gas is the ratio of its mass to its volume, and hence it follows that the pressure a gas exerts varies inversely with its volume, if its mass or the number of particles in a given volume remain the same; and this result is the same as that previously obtained by experiment, and known as Boyle's law.

The notion that a gas consists of a series of particles flying about in all directions is the basis of what is called the kinetic theory of gases. This theory, besides explaining the phenomena of diffusion and Boyle's law, accounts for many other facts connected with the action of gases at different temperatures; but this subject is beyond the range of the present volume.

CHAPTER VII.

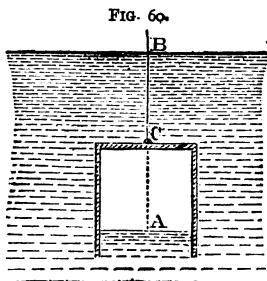
PNEUMATIC INSTRUMENTS.

XVII. *Diving Bell. Pressure Gauges.*

THE two principles which we have now established—viz., that the air is a heavy elastic fluid, and that the density of a gas varies with its pressure—serve to explain most phenomena exhibited by gases at constant temperature, and enable us to understand the action of a great variety of pneumatic instruments.

§ 109. **The Diving Bell.**—If we take an ordinary glass tumbler and immerse it vertically, with its mouth downwards, in a tub of water, we shall find that the enclosed air will prevent the water from rising in the inside except to a very small height, which will vary with the depth to which the tumbler is immersed. The action of the diving-bell is similar. It consists of a hollow vessel, nearly cylindrical in form, and open at its lower end. When lowered vertically into water the enclosed air is compressed by the weight of water above it, and the water rises in the bell to a height which increases with the depth of the bell from the surface of the water. The bell is let down by a chain; and in order that men may be able to work in the inside, air is introduced by means of a pump through an opening in the top, and the pressure of the air prevents the water from rising in the bell.

§ 110. **Problems on the Diving Bell.**—(1.) To find the height to which water rises in the bell, at a given depth, when no additional air is introduced :—



Let $BC = z$, be the depth of the top of the bell below the surface of the water. Let b be the height of the bell, and H the atmospheric pressure at the surface of the water measured by a water barometer.

Then, if $CA = x$, the part of the bell occupied by the air at the depth z , and if we suppose the bell to be of uniform area inside, the pressure on the air within the bell is equivalent to the weight of a column of water the height of which is $H + BA = H + z + x$. Hence, by Boyle's law :—

$$\frac{x}{b} = \frac{H}{H+z+x} \text{ or, } x^2 + x(H+z) = Hb$$

a quadratic equation, the positive solution of which gives the height required.

(2.) To find the volume of air at the ordinary atmospheric pressure that must be introduced into the bell at a given depth, to prevent any water from entering :—

Let z be the depth of the top of the bell, and b the height of the bell, as before. Then, if V be the volume of the air in the bell at the normal pressure H , and V' the volume which the compressed air in the bell at depth $z + b$ would occupy at pressure H , it follows

from Boyle's law that

$$\frac{V'}{V} = \frac{H+z+b}{H}$$

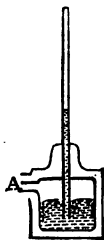
$$\therefore V' - V = \frac{z+b}{H} \cdot V$$

i.e. the volume of air introduced, at the ordinary atmospheric pressure, is $\frac{z+b}{H} \cdot V$.

§ 111. **Manometers, or Pressure-Gauges.**—Manometers are instruments for measuring the elastic force of a gas contained in a closed vessel.

FIG. 61.

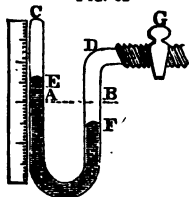
The simplest form of manometer consists of a long narrow open tube which dips into a strong box containing mercury. The gas, the pressure of which is to be measured, is admitted into the box through an opening, A, and if the elasticity of the gas is equal to that of the air the level of the mercury in the tube will be the same as that in the box. If, however, the elasticity of the gas is greater, the mercury will be forced up the tube, and the excess of the pressure of the gas over that of the atmosphere can be measured by the height of the mercury in the tube.



This form of manometer cannot be used for very great pressures, as the length of the tube renders it inconvenient. Thus, for a pressure of two atmospheres the tube must be 30 inches, and the length of the tube must be increased 30 inches for every additional pressure of one atmosphere that is to be measured.

§ 112. **Compressed Air Manometer.**—For measuring greater pressures this form of instrument is better suited.

FIG. 62



It consists of a bent tube, one branch of which is closed and contains air at the ordinary atmospheric pressure. This air is shut off from the other branch by some mercury which occupies the lower part of the tube. The open branch communicates with the vessel containing the gas, the pressure of which is to be measured. The closed branch of the tube is furnished with a scale.

If the mercury stands at the same level in both branches of the tube the pressure of the gas will equal that of the atmosphere. But if a gas or vapour of greater pressure be admitted through G the level of the mercury will fall in D and rise in C above the original level, A B.

The measure of the pressure of the gas in D is that of the compressed air in C, together with that indicated by the difference of level of the mercury in the two branches of the tube.

In order to graduate the scale we must find the distance A E or *rise* of the mercury corresponding to a pressure of n atmospheres in D F. Let A C, the space originally occupied by the air, equal a , and let $A E = x$.

Let H be the height of the mercury in the barometer at the atmospheric pressure, and p the pressure of the air in C E.

Then $p \times CE = H \times AC$, or $\frac{p}{H} = \frac{AC}{CE} = \frac{a}{a-x}$ and the pressure of the gas in $DF = p + 2EA = nH$.

$$\therefore nH = H \cdot \frac{a}{a-x} + 2x$$

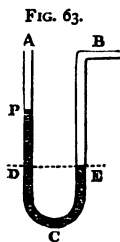
$\therefore 2x^2 - (nH + 2a)x + (n-1)Ha = 0$;
and, by the solution of this quadratic, the value of x , corresponding to any number of atmospheres, can be determined and the scale graduated.

§ 113. **The Siphon Gauge.**—For measuring small pressures this instrument is sometimes employed. It consists of a bent tube, open at both ends, one of which communicates with the vessel containing the gas. The liquid used is water or mercury.

If the gas is admitted through B, and the liquid assumes a difference of level, PD , then the pressure of the gas equals the atmospheric pressure + the weight of the liquid in PD . If, however, the liquid rises in the other branch of the tube, then the pressure of the gas = the pressure of the atmosphere, *minus* that due to the difference of level.

Thus, if the liquid fall through x inches in one branch, it will rise through x inches in the other; and the difference of level will be $2x$. If, therefore, a be the sectional area of the tube and s the specific gravity of the fluid—

The pressure of the gas = atmospheric pressure $\pm 2xs$, according as the level sinks in B or A.



EXERCISES. XI.

1. To what depth must the top of a diving-bell 8 ft. high be immersed under water that the air may be compressed to half its volume, the height of the water barometer (H) being equal to 34 ft. ?
2. What additional volume of air at the ordinary pressure ($H = 34$ ft.) must be admitted into a bell 8 feet high, the internal section of which is uniform and equal to 20 sq. ft., and the top 60 feet below the surface, to completely fill it ?
3. A cylindrical bell, the height of which is 6 feet, is furnished with a barometer that stands at 30 in., and is lowered into water till the barometer stands at 40 in. : find the depth of the top of the bell below the surface of the water ; sp. gr. of mercury = 13.6.
4. A barometer marking 30 in. is carried down in a diving-bell which is kept constantly full of air : find the depth of the top of the bell from the surface of the water, when the barometer marks 42 in., the height of the bell being 8 feet.
5. A gas, the pressure of which is 10 lbs. on a square inch, communicates with a siphon gauge (fig. 63), the section of which is 1 square inch : find the difference of level, supposing the instrument contains mercury and the ordinary atmospheric pressure is 15 lbs. on the square inch.
6. If in the open manometer the distance of the level of the mercury from the top of the box is a , and a gas be admitted that depresses the mercury b within the box, find the height of the mercury in the tube above the original level, the atmospheric pressure being h .
7. A cylindrical bell, 4 feet deep, whose interior volume is 20 cubic feet, is lowered into water until its top is 14 feet below the surface of the water, and air is forced into it until it is three-quarters full. What volume would the air occupy under the atmospheric pressure, the water barometer being at 34 feet ?
8. Find the depth to which a cylindrical diving-bell 8 feet

high must be sunk in water in order that the water may rise in it 3 ft. ($H = 34$ ft.)

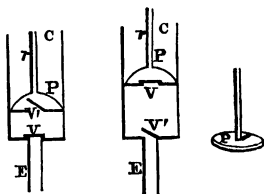
9. If the weight of a cylindrical bell is 2,000 kils. and the specific gravity of the material 8, find the tension in the chain when the bell is immersed to such a depth that the pressure of the enclosed air equals 3 atmospheres, the interior volume being 1.5 cubic meters.

XVIII. Air-Pumps.

§ 114. **Essential Parts of a Pump.**—A pump is an instrument for removing a fluid from a reservoir, or vessel containing it, by the forcible withdrawal of the atmospheric pressure.

It consists essentially of (1) a barrel or *cylinder*, C , through which the fluid escapes; (2) a disc or *piston*, P , capable of moving up and down the cylinder, into which it exactly fits, and worked by a handle or *rod*, r , attached to it; (3) a *pipe*, E , that communicates with the reservoir or vessel from which the fluid is to be removed; and (4) valves

FIG. 64.

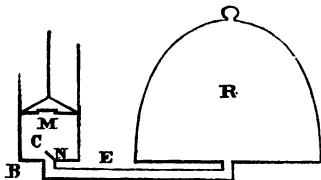


or small apertures, with movable covers, opening one way only, v, v' , which serve to admit the fluid from one part of the instrument to another, and to prevent it from returning. A valve is generally found at the end of the pipe, where it communicates with the cylinder, and very frequently in the piston itself.

§ 115. **The Air-Pump.**—This is an instrument for removing air from a closed vessel.

It consists of a *receiver*, R, from which the air is to be removed ; of a cylinder or barrel, A B, furnished with a piston and valves ; and of a pipe which serves to connect the receiver with the cylinder.

FIG. 65.



The action of the pump may be thus explained:—

Suppose the piston at B, when the receiver, R, is full of air. As the piston is raised the valve M, which opens upwards, remains closed in consequence of the pressure of the external air, and the air from R rushes through the pipe E, opens the valve N, and occupies the space between the piston and the bottom of the cylinder. When the piston is first pushed down the air in the cylinder is compressed, the valve M remaining closed ; but as soon as the pressure of the air in the cylinder begins to exceed that of the air outside the valve M is opened, and the enclosed air escapes as the piston descends through it. The piston being again raised, the same process is repeated. It should be noted that it is only when the piston is raised that air is withdrawn from the receiver, and that when it descends the air so withdrawn escapes to the outer atmosphere.

§ 116. To determine the density of the air in the receiver after any number of strokes.

Let V and v be the volumes of the receiver and cylinder respectively. Let d be the original density

of the air in R, when the piston is at B. Then Vd equals the mass of air in R. When the piston rises to A the mass of air in R occupies the space $V+v$; and if d_1 be its diminished density we have

$$Vd = (V + v) d_1, \text{ or } d_1 = \frac{V}{V + v} \cdot d,$$

since the mass remains the same.

When the piston descends to B a part of the air escapes, and the mass of air in R = Vd_1 . If now d_2 represent the density of the air when its volume increases to $V + v$, we have

$$Vd_1 = (V + v) d_2, \text{ or } d_2 = \left(\frac{V}{V + v} \right)^2 d,$$

and similarly if d_3 represent the density of the air in R after three complete strokes—

$$Vd_2 = (V + v) d_3, \text{ or } d_3 = \left(\frac{V}{V + v} \right)^3 d,$$

and, consequently if d_n be the density of the air in the receiver after n complete or double strokes

$$d_n = \left(\frac{V}{V + v} \right)^n d.$$

It will be seen that no amount of exhaustion can reduce the density of the air to zero. At each stroke of the piston a fraction only of the air in the receiver is removed; and as the remaining air occupies the whole volume of the receiver, a perfect vacuum cannot possibly be obtained.

§ 117. **Examples.**—(1.) If the volume of the receiver is 64 cubic inches and of the cylinder 8 cubic inches, what quan-

tity of air would be left in the receiver after two complete strokes?

At the first double stroke 8 cubic inches of air at the original density would be removed and 56 would remain. Of these one-eighth, or 7 cubic inches, would be removed at the second stroke, and consequently 49 would be left.

(2.) After four complete strokes the density of the air in the receiver is to its original density as 10,000 : 14,641 : compare the volumes of the receiver and cylinder.

$$\text{Here } \frac{d_1}{d} = \frac{10,000}{14,641} = \left(\frac{V}{V+v} \right)^4$$

$$\therefore \frac{V}{V+v} = \sqrt[4]{\frac{10,000}{14,641}} = \frac{10}{11} \quad \therefore V : v :: 10 : 1.$$

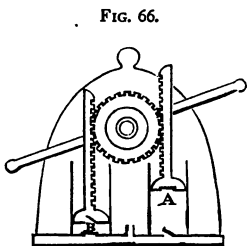
§ 118. **Difficulty of Working.**—It is to be observed that the difficulty of working this kind of air-pump increases with the number of strokes. For, neglecting the frictional resistance, the force required to raise the piston depends on the difference of pressure at the upper and lower surfaces of the piston. Now, this difference increases with the rarefaction of the air in the receiver, and consequently the difficulty of working the machine increases as the exhaustion proceeds.

In lowering the piston the external pressure assists the action, so long as the air in *c* is of less density than that outside ; but as soon as that point has been reached an expenditure of force is necessary to overcome the inside pressure and open the valve *m*. Thus the difficulty of lowering the piston decreases somewhat with the number of strokes.

§ 119. **The Double-barrelled Air-Pump.**—This instrument, known as Hawksbee's air-pump, has two

cylinders, in each of which is a piston worked by a rack and pinion. To the centre of the toothed-wheel is fixed a handle, by moving which to and fro the pistons are made alternately to ascend and descend.

The chief advantage of this instrument over the one already described is that a volume of air equal to that of the cylinder is removed at each single stroke of the piston, and that, consequently, the rate of exhausting the receiver is doubled.

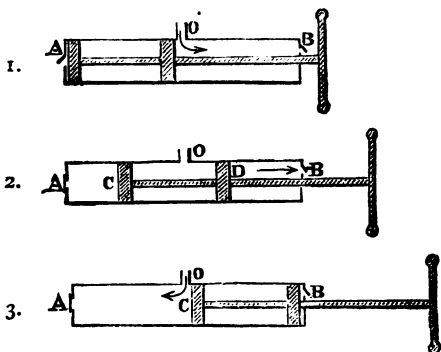


In the adjoining figure the piston A is ascending and withdrawing air from the receiver, and the piston B is descending and discharging into the outer atmosphere the air previously withdrawn from the receiver. The position of the valves should be carefully noted.

Another advantage of this machine is that it is easier to work. The difficulty of raising one piston is compensated by the assistance which the pressure of the atmosphere affords in forcing down the other piston, and thus the difficulty of working the machine does not increase, as in the case of the single-barrelled air-pump, with the number of strokes. In pressing down the piston no force is required till the air beneath the piston has been compressed to the density of the air outside; and this occurs nearer and nearer to the bottom of the cylinder as the degree of exhaustion increases. Consequently, the instrument is worked somewhat more easily as exhaustion proceeds.

§ 120. **Tate's Air-Pump.**—This instrument combines the advantage of double action with a single barrel. The barrel is generally horizontal, and communicates

FIG. 67.



with the receiver by a vertical opening, *O*. The piston, *C D*, occupies a little less than half the length of the barrel, and consists generally of two discs rigidly connected together by the piston-rod which unites both.

The principle of the action would be the same if the piston were solid and of uniform area throughout; but the trouble of working it would be greater. The barrel is furnished with two valves, *A* and *B*, at either end of it, opening outwards.

The action may be thus explained :—

Suppose the piston to be, first of all, in the position shown in fig. 1, all the air in front of it having been expelled through *A* by the driving of the piston home. If the piston be now pulled out, as in

fig. 2, the valve A will close, and the air in front of the piston will pass through B, which will remain open. When the piston reaches the farther end, B, as shown in fig. 3, air will escape from the receiver through o into the empty space left behind the piston. When the piston is pushed inwards this air will be expelled through A; and on the piston reaching A, as shown in fig. 1, the empty space behind it will be again occupied by the air from the receiver. In this way, a certain volume of air, equal to about half the contents of the barrel, will be removed at each stroke of the piston; and, as the difference of pressure at the two ends of the piston decreases with every stroke, the working of the pump becomes easier as the exhaustion proceeds.

§ 121. **Mercury Gauge.**—The pressure of the air, after any number of strokes, in the receiver of an air-pump is indicated by a gauge, which is generally attached to the connecting-pipe of the air-pump.

In its simplest form it consists of a straight tube, open at both ends. The upper end is connected with the receiver, and the lower end dips into a cup of mercury. As the air is removed from the receiver the pressure inside the tube is less than that on the mercury in the cup, and consequently the mercury rises in the tube. This instrument enables us to watch the process of exhaustion from the very first stroke. If the barometer at the time marks 30 inches, and the mercury has risen 4 inches in the tube, the pressure

FIG. 68.



of the air in the receiver is $30 - 4 = 26$ in. The necessary length of this form of gauge renders it somewhat inconvenient.

The more commonly employed gauge consists of a bent tube (fig. 69), having one end closed and the other

FIG. 69.



FIG. 70.



open. Each branch is about ten inches in length, and the closed end is filled with mercury, the weight of which is supported by the atmospheric pressure. The instrument is enclosed in a glass case, which communicates with the receiver of the air-pump. The first few strokes do not produce any change in the gauge, but as

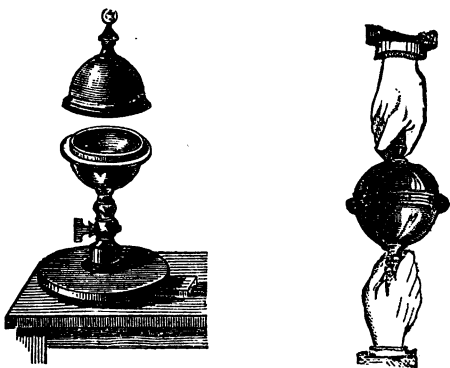
soon as the tension of the air in the receiver is less than the pressure due to the column of mercury in the closed end of the tube the mercury begins to fall, and the difference of level of the mercury in the two ends measures the pressure of the air in the receiver. The open end is sometimes bent again upon itself, as in fig. 70, and screwed into the connecting pipe.

§ 122. **Experiments.**—The experiments that can be performed with the air-pump are very numerous. We have found it necessary to refer to some of them, in order to prove that the air has weight (§ 90). The following additional experiment should also be performed :—Take a hollow cylindrical vessel and stretch a bladder over one end, and then place it on the plate of the air-pump, having carefully greased the edges of the glass vessel, so as to prevent the entrance

of the air. As the air from under the bladder is gradually removed the bladder will be found to yield under the pressure of the external air, and will at length break.

By means of the Magdeburg hemispheres, invented by Guericke, some idea may be formed of the magnitude of the pressure which the atmosphere exerts on a comparatively small surface. Their gene-

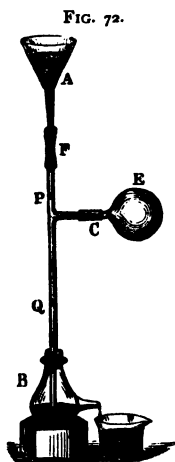
FIG. 71.



ral form is shown in fig. 71. The edges are greased and pressed together, and the enclosed air is then removed by the air-pump. In order to separate the two hemispheres by pulling them asunder, a very great effort must be made ; and this will be found to be the case in whatever position the hemispheres may be held, showing also that the atmospheric pressure acts equally in all directions.

§ 123. **Sprengel's Air-Pump.**—By means of the following contrivance a small receiver can be more effectually exhausted than by any of the pumps already described.

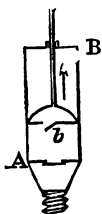
It consists of a tube, *FB*, open at both ends, and fitted to a funnel *A*, by a piece of indiarubber. The



funnel contains mercury, the flow of which through the tube can be regulated by tightening the indiarubber connection. The tube is considerably longer than a barometer tube, and has a spout in its side attached to the receiver, *E*, which is to be exhausted. The lower end of the tube, *B*, dips into a vessel of mercury. As soon as the mercury begins to flow exhaustion commences. The first drops of mercury that run out close the lower end of the tube and prevent the air from entering. As each drop of mercury passes the neck, *c*, the air in the receiver, *E*, expands and occupies the space *cQ*. In this way the tube becomes filled with cylinders of air and mercury, which gradually escape from the spout into the vessel of mercury. The mercury is poured again into the funnel, *A*, and the process is repeated till the tube is occupied by a continuous column of mercury, the height of which is equal to that of the mercury barometer. The exhaustion is now complete, and the receiver, *E*, corresponds to the Torricellian vacuum of the ordinary barometer.

§ 124. **The Condensing Syringe.**—This consists of a barrel, A B, into which an air-tight piston fits, having in it a valve that opens downwards. At the bottom of the barrel is a valve that likewise opens downwards and communicates with a receiver, to which the syringe is tightly screwed. When the piston is moved down, the valve *b* is closed and the valve A opened by the increased pressure. As the piston returns, the pressure of the air in the receiver closes the valve A, and thus prevents the air from re-entering the barrel. It is evident that the same quantity of air will enter the receiver at each stroke of the piston, if the machine works perfectly and the piston is moved through the whole length of the barrel.

FIG. 73.



EXERCISES. XII.

1. If the volume of the receiver of an air-pump be eight times that of the barrel, compare the density of the air after the third stroke with its original density.
2. If the receiver of an air-pump holds 90 grains of air at the ordinary pressure, and if the barrel can hold 10 grains, what will be the weight of the air in the receiver after four complete strokes?
3. If one-third of the contents of the receiver is removed at each complete stroke, and the barometer stand at 75 cm., find the height to which the mercury will rise in the simple barometer gauge after three strokes.
4. The mercury rises in the barometer gauge through $2\frac{1}{2}$ in. in two complete strokes: compare the size of the barrel with that of the receiver ($h = 30$).

5. The branches of a siphon barometer gauge are each 16 cm. long, and the closed branch is filled with mercury, which also occupies 1 cm. of the open branch: compare the density of the air in the receiver with its original density, when the mercury begins to fall in the siphon-gauge ($h = 75$ cm.).

6. A receiver attached to an air-pump has the volume of 100 cubic inches, while the cylinder has the volume of 10 cubic inches. What proportion of the original air will be left in the receiver after the completion of the fourth double stroke?

7. The capacity of the barrel of a condensing air-pump is 10 cubic inches, and of a copper receiver 100 cubic inches. By how much will the pressure of the air in the receiver be increased after 20 strokes of the piston?

8. When the height of the barometer is 75 cm., the air in the receiver of an air-pump is exhausted until the mercury in the barometer-gauge (fig. 68) attached to it rises from 0 to 36 cm. By how much has the tension of the enclosed air been reduced?

XIX. *Pumps for Liquids.*

§ 125. **Common or Suction Pump.**—This is an instrument for drawing water from a well or subterranean reservoir.

It consists of a cylinder fitted with a piston and valve, and connected by a second valve with a pipe which communicates with the reservoir. The cylinder MN is called the pump-barrel; the tube ve the suction-tube. The mode of action is as follows:—

Suppose the piston at first to be at N , and the suction-tube filled with air at the atmospheric pressure. If the piston be raised the air in $N D$ will expand, open the valve v , and follow the piston. At the same time, since the pressure on the surface of the water

within the tube is diminished, owing to the expansion of the contained air, the external pressure at D will cause the water to rise in E to such a height that *the elasticity of the air below A B, together with the weight of the column of water above D in the suction-tube, equals the atmospheric pressure without.*

As the piston descends the air below it is compressed, and escapes after a time through the valve F, the valve v being closed by the increased pressure of the air above it. Thus the water remains at the same level in the suction-tube whilst the piston is descending.

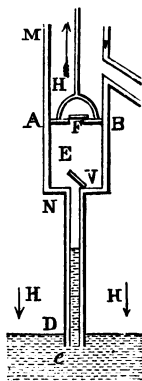
When the piston is again raised the pressure is removed from above v, and the air underneath it at once opens the valve and occupies the space beneath A B, the water rising in the suction-tube as before.

This action continues till the water has risen to N, when the raising of the piston causes the water to enter the barrel, provided the height D N is less than H , the height of the water barometer. As the piston continues to rise the water will follow it so long as the height of the piston above the level of the water outside is less than H .

As the piston descends, the pressure of the water beneath it opens the valve F, and the piston passes through the water. When the piston again ascends the water is discharged at the spout, and the barrel is refilled through the suction-tube.

The water having once entered the barrel, the

FIG. 74.



contents of the barrel are discharged at each upward stroke of the piston ; but, in order that a volume of water equal to that of the barrel may be discharged at each upward stroke, it is necessary that the atmospheric pressure should first raise the water to the spout ;—*i.e.* the height of the spout above the level of the water outside must be less than H .

It is evident that if the piston in descending does not reach v , so as ultimately to exclude all air from underneath it, the water may never be able to enter the pump-barrel, even if ND is less than H .

It is not essential to the working of the pump that the tube should be straight ; nor does it matter at what horizontal distance from the pump-barrel the suction-tube enters the reservoir.

§ 126. Force required to raise the piston-rod.

First. Suppose the water has not yet entered the barrel. In this case the force necessary to raise the piston-rod is equal to the difference of the pressure of the air on the top and bottom of the piston AB (fig. 74). Let E represent the pressure of the air below AB measured in water, and let H be the height of the water barometer outside. Let z equal the height of the column of water in the tube.

Then $H = E + z$; and the force required to raise the piston is

$$AB \times (H - E) = AB \times z$$

= the weight of a column of water that has AB for base, and the distance of surface-level of water in the pump above the level outside for height.

Secondly. Suppose the barrel already full of water. The force required is, as before, the difference of pressure on the two sides of the piston.

Let $x = c A$ (fig. 75) be the height of the water above the piston at any instant ; then the pressure on the upper surface of the piston is

$$(H+x) \times A B$$

and the pressure on the lower surface is

$$(H-z) \times A B$$

where z is, as before, the height of the water below the piston above the level of the reservoir. Hence, the difference, or force required is

$$A B \times (x+z),$$

or the weight of the column of water having $A B$ for base and the difference of the level of the water in the pump and in the reservoir for height. This, therefore, is the measure of the tension of the piston-rod in all cases.

When the pump is in full action, discharging at each stroke a volume of water equal to that of the barrel, the tension of the piston-rod is constant.

§ 127. **Examples.** (1.) The length of the suction-tube of a common pump is 12 feet, and the piston when at its lowest point is 2 feet from the fixed valve ; if at the first stroke the water rises 11 feet in the tube, find the extreme length of the stroke, supposing the water barometer to stand at 33 feet, and the area of the barrel to be three times that of the tube.

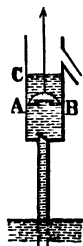
If x be the length of the stroke, and a the area of the tube, the volume originally occupied by the air is $12a + 2 \times 3a = 18a$; the volume occupied by the air after the first stroke is

$$3a(x+2) + a = (3x+7)a.$$

$$\therefore \text{by Boyle's law, } \frac{18}{3x+7} = \frac{33-11}{3} = \frac{2}{3}$$

$$\text{or, } x = 6\frac{2}{3} \text{ feet.}$$

FIG. 75.



(2.) The suction-tube of a common pump is 12 feet long, and the piston, starting from the fixed valve, is raised at the first stroke through 3 feet. If the area of the barrel is four times that of the tube, find the height to which the water will be raised in the tube ($H = 34$ feet).

Let x be the required height, a the area of suction-tube. The volume originally occupied by the air was $12a$. After the first stroke the air occupies $(12-x)a + 3 \times 4a = (24-x)a$; and the pressure is reduced from 34 to $34-x$ feet.

Hence, by Boyle's law,

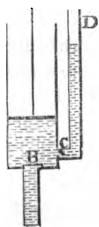
$$\frac{12}{24-x} = \frac{34-x}{34}, \text{ or } x = 8.2 \text{ feet nearly}$$

§ 128. **The Lifting Pump.**—This is a modification of the common pump, in which the water discharged from the pump-barrel, which is closed at the top, enters a pipe furnished with a valve and communicating with the spout.

The action is the same as before; but the water, instead of flowing away through the spout, is lifted into the pipe and prevented from returning by a valve. In this way water can be stored up at any elevation, and can afterwards be made to flow through the spout as required.

§ 129. **The Forcing Pump.**—In this pump the piston has no valve. The action is the same as in

FIG. 76.



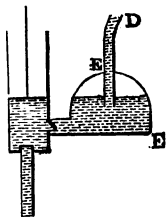
the common pump until the water enters the barrel. Then, as the piston is raised the water occupies the space beneath it, and passing through the valve c , rises in the tube CD to the same level as in the barrel.

As the piston descends the valve B is closed, and all the water contained in the barrel is *forced* up the pipe and prevented

from returning by the valve *c*. In this way water can be forced up to any height consistent with the strength of the material, or can be made to rise in a jet from the upper end of the pipe. No flow, however, occurs during the ascent of the piston.

§ 130. **Forcing Pump, with Air-vessel.**—To obtain a continuous stream of water from the top of the pipe *c d* (fig. 76) the water must be first admitted into a strong vessel containing air. The action being the same as before, the piston in descending forces the water into the vessel *e f* (fig. 77), and compresses the air in the upper part of it. As more water is forced into the vessel the air is further compressed, and

FIG. 77.



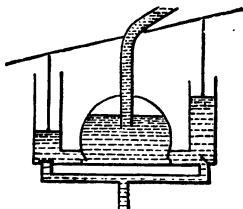
the water rises in the tube *e d* and flows out from *d*. When the piston is drawn up the flow from *d* would cease but for the fact that the air in *e f*, freed from its former pressure, now expands and forces the water up the tube, thus causing an unbroken flow from *d*.

The elasticity of the air in *e f* will decrease with the escape of water from *d*, and consequently the pressure of the water in the pipe must never be greater than the excess of the pressure of the air in *e f* over the ordinary atmospheric pressure. If the height of the pipe is inconsiderable, the continuity of flow can be easily preserved during the ascent of the piston.

§ 131. **The Fire Engine.**—This is a double forcing pump connected with an air-chamber. The constancy of flow is obtained not only by the air-vessel, but also by the alternate action of the two pumps. The pistons

are worked by a lever, so that one ascends while the other descends. Every time the pistons momentarily

FIG. 78



stop, the elasticity of the air in the air-chamber maintains the flow.

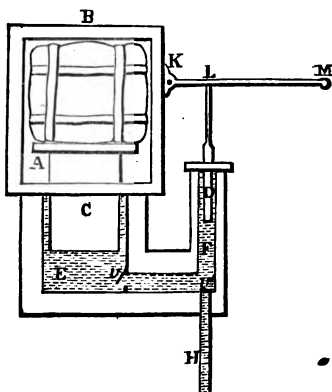
§ 132. **Bramah's Press.**—This is a practical application of Pascal's principle of the equal transmissibility of fluid pressure, and consists of an apparatus very similar to that explained in § 27, with the substitution of a forcing-pump for the weighted piston.

In fig. 79, A is a platform which supports the substance to be pressed against the strong framework B. C and D are two solid cylinders which serve as pistons, and E and F are two hollow cylinders connected by a pipe furnished with a valve *v*.

The vessel F communicates with a reservoir of water by the pipe H, so that D F H constitutes an ordinary forcing pump. The piston D is attached to a lever, K L M, and the power is applied at M.

When the instrument is in action the vessels E and F are filled with water, and as the piston or plunger D descends, it closes the valve *v'* and forces the water into the vessel E, and raises the cylinder C, with its attached platform.

FIG. 79.



If Q be the pressure produced by the plunger D in its descent, this pressure is distributed equally over the surface of the cylinder c ; and if A be the area of c , and a that of D , then the pressure exerted at c is $\frac{A}{a}Q$.

If we suppose P to be the force that must be applied at M , to produce the pressure Q at L , then it follows from the principle of the lever, that

$$Q : P :: MK : LK$$

$$\text{or, } Q = \frac{MK}{LK} \cdot P$$

If, therefore, W represent the force with which the cylinder, whose area is A , is pressed upwards, we have

$$W = \frac{A}{a} \cdot Q = \frac{A}{a} \cdot \frac{MK}{LK} \cdot P; \text{ and if } R \text{ and } r \text{ are the}$$

radii of the two cylinders respectively, and $M K = l$ and $L K = c$, we have

$$\frac{W}{P} = \frac{R^2}{r^2} \cdot \frac{l}{c}$$

This instrument is very extensively employed where the application of a considerable pressure through a small space is required. The very great pressure to which the water is subjected necessitates great care in the construction of the collars through which the cylinders work, so as to render them perfectly water-tight.

EXERCISES. XIII.

1. The spout of a common pump is 16 feet from the surface of the water in the reservoir. The area of the pump-barrel is 72 square inches: find the tension of the piston-rod when the pump is full of water.

2. The specific gravity of mercury is 13.6, and the height of the mercurial barometer is 30 inches. What is the greatest height to which water can be raised by means of the common pump?

3. Find the pressure that can be produced by a Bramah's press if the areas of the pistons are 8 : 1, and if a force of 10 lbs. is applied at the end of a lever 2 feet long, and at a distance of 20 ins. from the point where the piston-rod is attached to it.

4. If in the lifting pump the area of the barrel is four times as great as that of the pipe, compare the pressures on the two valves at the commencement of the third stroke after the water has entered the pipe.

5. If the area of the barrel of the forcing pump be 10 square inches, and of the pipe into which the water is forced 2 square inches, find the height to which water can be raised in three complete strokes, supposing the piston-range to be 1 foot.

6. Find the force that must be applied to the piston-rod, in the preceding question, at the beginning of the third downward stroke.

7. If the diameter of the piston of a common pump be 4 inches, and the height of the head of the water in the pump 18 feet above the well, find what pressure the piston bears, taking the weight of 1 cubic foot of water equal to 62.3 lbs.

8. The length of the suction-tube is 20 feet, and the entire stroke of the piston is 6 feet. The piston starts from the bottom of the barrel, and at the end of the first stroke the water has risen 12 feet in the tube. Compare the area of the barrel with that of the tube ($H = 34$ feet).

9. A small strong pump is employed for raising mercury from a vessel. The height of the fixed valve is 2 feet above the level of the mercury in the vessel, and the spout is 8 in. above the valve. When the pump is in full action what part of the contents of the barrel can be ejected at each stroke of the piston ($H = 30$ in.)?

10. A suction-tube is used for drawing up mercury from a vessel containing it, and the piston is raised 10 inches from its lowest point, which is 2 inches from the valve at the bottom of the tube. To what height will the mercury rise in the tube ($H = 30$ in.)?

MISCELLANEOUS PROBLEMS.

1. A CUBE of brass, whose edge is 2 inches and specific gravity 8, is completely imbedded in a cube of wood, whose edge is 3 inches and specific gravity 5. Find the mean specific gravity of the whole cube.

2. When equal volumes of alcohol (specific gravity = 0.8) and distilled water are mixed together, the volume of the mixture (after it has returned to its original temperature) is found to fall short of the sum of the volumes of its constituents by 4 per cent. Find the specific gravity of the mixture. (Univ. Lond. Matric.)

3. The specific gravity of cast copper is 8.88, and that of copper wire is 8.79. What change of volume does a kilogramme of cast copper undergo in being drawn out into wire? (Univ. Lond., 1st B. Sc.)

4. A mixture is made of 7 cubic centimetres of sulphuric acid (specific gravity = 1.843) and 3 cubic centimetres of distilled water, and its specific gravity when cold is found to be 1.615. Determine the contraction which has taken place. (1st B. Sc. 1874.)

5. Two liquids are mixed (1) by volume in the proportion of 1 : 4, and (2) by weight in the proportion of 4 : 1. The resulting specific gravities are 2 and 3 respectively. Find the specific gravities of the liquids.

6. The specific gravity of a mixture of two different liquids being supposed to be an arithmetic mean between those of the component liquids; required the ratio of the volumes of the latter contained in the mixture. (Univ. Lond., B.A. 1877.)

7. Find the pressure on a vertical rectangle, 10 inches long and 6 inches broad, immersed in water with its longer sides horizontal and with the upper one 2 inches below the surface. (One cubic foot of water weighs 1,000 ounces.) (Matric. 1877.)

8. A vessel in the shape of a pyramid, 5 feet high, and with a base 4 feet square, is filled with water. Find the pressure upon the base, and account for its being greater than the total weight in the vessel.

9. Find the whole pressure on the lower half of the curved surface of a vertical cylinder filled with water, the area of the base being $12\frac{1}{2}$ square cm. and the height 1·4 decim.

10. A piston, 6 square inches in area, is inserted into one side of a closed cubical vessel, measuring 10 feet each way, filled with water: the piston is pressed inwards with a force of 12 lbs. Find the increase of pressure produced on the entire surface of the vessel. (B. A. 1872.)

11. The pressure at the bottom of a well is four times that at the depth of 2 feet; what is the depth of the well if the pressure of the atmosphere is equivalent to 30 feet of water? (Camb. Gen. Exam. 1877.)

12. A and B are vessels full of water, with circular and horizontal bases, 12 inches and 8 inches in diameter respectively. A is 8 inches, and B is 9 inches high. Compare the pressure on the bases. (Cam. Gen. Exam.)

13. If a piece of wood weighing 120 lbs. floats in water with four-fifths of its volume immersed, show what is its whole volume (1 cubic foot of water weighs 62·5 lbs.). (Matric. 1872.)

14. A wine-bottle, which below the neck is perfectly

cylindrical and has a flat bottom, is placed in pure water. It is found to float upright, with $4\frac{1}{2}$ inches immersed. The bottle is now removed from the water and put into oil, the specific gravity of which is 0.915. How much of it will be immersed in the latter fluid? (Matric. 1874.)

15. Two pieces of iron (specific gravity 7.7) suspended from the two scale-pans of a balance, the one in water and the other in alcohol of the specific gravity 0.85, are found to weigh exactly alike. Find the proportion between their true weights. (B. Sc. 1875.)

16. An inch cube of a substance of specific gravity 1.2 is immersed in a vessel containing two fluids which do not mix. The specific gravities of these fluids are 1.0 and 1.5. Find what will be the point at which the solid will rest. (Matric. 1876.)

17. A substance which weighs 14 lbs. in air and 12 lbs in water, floats in mercury whose density is 13.6. What proportion of its volume will be immersed?

18. A solid, of which the volume is 1.6 cubic centimetres, weighs 3.4 grams in a fluid of specific gravity 0.85. Find the specific gravity and weight of the substance. (B. Sc. 1876.)

19. An accurate balance is totally immersed in a vessel of water. In one scale-pan some glass (specific gravity 2.5) is being weighed, and exactly balances a one-pound weight (specific gravity 8.0), which is placed in the other scale-pan. Find the real weight of the glass. (Matric. 1875.)

20. A right cone, whose weight is W , floats in a liquid, vortex downwards, with $\frac{1}{3}$ of its axis immersed; what additional weight must be placed on the base of the cone so as just to sink it entirely in the liquid. (Woolwich Exam.)

21. A cube floats in distilled water under the pressure of the atmosphere, with four-fifths of its volume immersed and with two of its faces horizontal. When it is placed under a condenser where the pressure is that of ten atmospheres, find the

alteration in the depth of immersion (the specific gravity of air at the atmospheric pressure being .0013). (B. Sc.)

22. At the bottom of a mine a mercurial barometer stands at 77.4 centimetres; what would be the height of an oil barometer at the same place, the specific gravity of mercury being 13.596, and that of oil 0.9? (Matric. 1876.)

23. A certain quantity of air at atmospheric pressure has a volume of 2 cubic feet, the temperature being 55° Fahr. What does the volume of the air become when the pressure is increased by one-twentieth, the temperature meanwhile remaining the same? (Women's Exam. 1876.)

24. A syphon barometer is so constructed that the long closed tube has an internal sectional area equal to $\frac{1}{4}$ of an inch, while the short open tube has an internal sectional area equal to $\frac{1}{2}$ an inch. Find what fall will take place in the long tube of this barometer when the true pressure of the air falls one inch. (B. Sc. 1875.)

25. The mercury in a barometer stands at 30 inches; the section of the tube measures 1 square inch, and the vacuum above the mercury 6 cubic inches, as much air is passed up the tube as depresses the mercury to 29 inches: what would be the space occupied by the air under the atmospheric pressure? (B. Sc. 1872.)

26. In a tube of uniform bore a quantity of air is enclosed. What will be the length of this column of air under a pressure of three atmospheres, and what under a pressure of a third of an atmosphere, its length under the pressure of a single atmosphere being 12 inches? (B. A. 1876.)

27. A Mariotte's tube (fig. 51) has a uniform section of 1 square inch, and is graduated in inches, 6 cubic inches are enclosed in the shorter (closed) limb, when the mercury is at the same level in both tubes. What volume of mercury must be poured into the longer limb, in order to compress the air

into 2 inches? The barometer stands at 30 inches. (B. Sc. 1874.)

28. Two cubic centimetres of air are measured off at atmospheric pressure. When introduced into the vacuum of a barometer they depress the mercury which previously stood at 76 cm., and occupy a volume of 15 cubic centimetres. By how much has the mercurial column been depressed? (Matric. 1878.)

29. A tumbler full of air is placed mouth downwards under water, at such a depth that the surface of the water inside it is at a depth of $25\frac{1}{2}$ feet. Compare the weight of a cubic inch of air in the tumbler with that of a cubic inch of air outside—the barometer standing at 30 inches, and the specific gravity of mercury being 13.6.

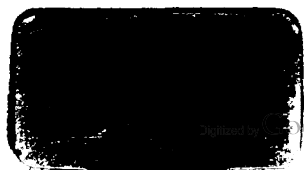
30. The contents of the receiver of an air-pump is six times that of the barrel. Find the elastic force of the air in the receiver at the end of the eighth stroke of the piston, when the atmospheric pressure is 15 lbs. to the square inch. (B. A. 1872.)

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